

**THE REPUBLIC OF TURKEY
BAHÇEŞEHİR UNIVERSITY**

**AN ADAPTIVE KANBAN CONTROL MECHANISM
SENSITIVE TO CHANGES IN INVENTORY**

Master's Thesis

ÖZGE ŞAHİN

İSTANBUL, 2013

**THE REPUBLIC OF TURKEY
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INDUSTRIAL ENGINEERING

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Supervisor: Asst. Prof. Dr. Barış Selçuk

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ÖZGE ŞAHİN

ABSTRACT

AN ADAPTIVE KANBAN CONTROL MECHANISM SENSITIVE TO CHANGES IN INVENTORY

ÖZGE, ŞAHİN

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There are a number of alternative production control approaches in literature. Most of them are adequate in theory but they are not applicable for a real life manufacturing environment. Uncertain demand and unbalanced production times are two important problems in a manufacturing environment. These two parameters affects the work-in-process and finished goods inventory level of the manufacturing system which leads to an increase in the total cost of manufacturing system. Therefore we utilized from a kanban operated Just-in-Time system and tried to adapt this concept into real life problems. We called our proposed system as an adaptive kanban control system. Our approach to developing an adaptive kanban system is different from the available techniques in the literature such that we explicitly incorporate the concept of update frequency. The number of kanban cards are updated (increased / decreased) according to the inventory status of the system. The new adaptive kanban control system is examined in two types. One of them includes a constant production time parameter. At the other one includes the production time parameter varies according to the work-in-process inventory status of the system. Our systems are modeled by continuous-time Markov chain and solved obviously. We specify our numerical results explicitly. Eventually, it is shown that our adaptive kanban system is superior to the traditional static kanban system under a prescribed set of conditions.

Keywords: Kanban, Adaptive kanban systems, Markov chains, Stochastic processes

ÖZET

ENVANTERDEKİ DEĞİŞİMLERE KARŞI DİNAMİK GÜNCELLENEN KANBAN TİPİ KONTROL MEKANİZMASI

ÖZGE, ŞAHİN

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Literatürde birçok alternatif üretim kontrol mekanizması yer almaktadır. Bunların bir çoğu teoride yeterlidir fakat gerçek bir üretim ortamına uygulanabilir değildir. Gerçek bir üretim ortamının iki önemli problemi belirsiz talep ve dengesiz üretim zamanlarıdır. Bu iki parametre üretim sisteminin toplam maliyetinde artışa yol açan yarı mamul ve son ürün envanter seviyelerini etkiler. Bu nedenle çalışmamızda kanban kontrollü tam zamanında üretim mekanizmasından yararlandık ve bu konsepti gerçek bir üretim ortamına adapte etmek üzerinde çalıştık. Önerilen sistemin adı uyarlanabilir kanban sistemleridir. Uyarlanabilir kanban sistemleri geliştirmekteki amacımızın literatürde mevcut olan çalışmalardan farkı güncelleme sıklığı konseptini kapsıyor olmasıdır. Sistemdeki kanban kartları sayısı sistemdeki envanter durumuna göre güncellenir. Yeni uyarlanabilir kanban kontrol sistemi iki şekilde incelenmiştir. İlkinde sistemin birim zamandaki üretim süresi parametresi sabittir. İkincisinde ise birim zamandaki üretim süresi parametresi sistemde bulunan yarı mamul envanter seviyesine göre değişmektedir. Sistemimiz Markov zinciri ile modellenmiş ve çözülmüştür. Sayısal sonuçlarımız açık bir şekilde gösterilmiştir. Son olarak uyarlanabilir kanban sistemi yaklaşımımızın belirli şartlar altında geleneksel kanban sistemleri yaklaşımına üstün olduğu açıkça gösterilmiştir.

Anahtar Kelimeler: Kanban, Uyarlanabilir Kanban sistemleri, Markov zinciri, Stokastik modelleme, Üretim

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ABBREVIATIONS

ACK	:	Acknowledgement
AWIP	:	Adaptive Work-in-Process
CONWIP	:	Constant Work-in-Process
FCFS	:	First-Come First-Served
FKS	:	Flexible Kanban System
JIT	:	Just-in-Time
MOP	:	Measure of Performance
MP	:	Manufacturing Process
MRP	:	Material Requirements Planning
SPT	:	Shortest Processing Time
TKS	:	Traditional Kanban System
WIP	:	Work-in-Process

SYMBOLS

Number of end products produced per unit of time	:	λ_p
Arrival of demand for end product per unit of time	:	λ_d
Production rate at Work-in-Process level n	:	$\lambda_{p(n)}$
Inventory level at the primal state of the manufacturing system	:	K
Maximum number of extra kanban cards	:	E_{max}
Kanban updating parameter	:	r
Service Level	:	SL
Fraction of production rate to demand rate	:	α
Finished goods inventory level at time t	:	$N(t)$
Number of active extra kanban cards in the system at time t	:	$X(t)$
Summation of steady-state probabilities for $j = 0, \dots, E_{max}$:	P_i
Steady-state probabilities	:	$P_{N(t), X(t)}$
Average finished goods inventory level	:	$E[I]$
Average Work-in-Process inventory level	:	$E[WIP]$
Average Lost Demand	:	$E[L]$
Number of identical parallel servers	:	b
Mean rate	:	μ

1. INTRODUCTION

Due to the recent evolutionary advancements in information technologies, monitoring the statuses of proprietary or outsourced facilities has become possible. Thus, revising and updating current plans or tactical control parameters based on the observed operational changes has become a popular practice. Usually, decision making with regard to updating depends on planners' individual assessments, and it is not done in a systematic approach. Although there are many different production control approaches today, many of them are not functional and they are not easy to implement. With the emergence of Just in Time (JIT) production control method, these issues are addressed. JIT is the most beneficial one of the production systems for the companies because as the name suggests, it responds to the demand of customer "Just in Time". Production is triggered with a customer demand in a JIT manufacturing environment. The production is done at the time needed, according to the type and amount of the demand. The benefit of such approach is improving efficiency and providing a high level of quality with eliminating all types of waste such as time, labor, and warehouse space. Based on this approach, the aim of JIT systems is holding almost zero level of inventory. It means that such a company can manage with their time and resources and allocate them very easily.

JIT approach is a kind of pull type control mechanism. A pull type control mechanism authorizes to manufacture a new item when there is a need for an end item. The manufacturing operations can be gathered into some stages in order to facilitate the flow of materials and information in a manufacturing system. Such kind of a system consists of one or more workstations. It is also triggered by the customer demand. Production will take place only if there is demand. When a demand arrives to the system, a demand signal is sent to the last stage of system and production starts. The demanded products are pulled through the system from the last workstation to the beginning to make sure the availability of other parts on the assembly track. The benefit of a pull type control mechanism is the shortened lead times.

1.1 KANBAN SYSTEMS

As we mentioned before when a demand arrives to a company, it operates as a pull type production control system, a signal is sent to the last station of system to trigger produc-

tion. This signal is a kanban card. A kanban operated JIT system is usually considered to be a typical pull system. Kanban is a Japanese word means ‘card’. A kanban is used as a production authorization card to ‘pull’ products through the system. The flow of parts throughout the product line is controlled by kanban cards (Kumar & Panneerselvam 2007). Kanban control systems are applied recently in manufacturing systems to efficiently manage the flow of materials and control the work-in-process inventory levels. A kanban card gives information about the type, time and amount of product required to be produced or assembled. Figure 1.1 shows an example of kanban card. There are some informations written on the kanban cards. These are the name and number of the part, identification of the part, the name or number of workstation that the card is used for, the kanban card number, the container number that the card is attached on and the description of the workstation where the kanban card will be released.



Figure 1.1: An example of a Kanban Card

There are two types of kanban cards in a traditional kanban control system. One of them is ‘one-card system’. In a one-card system when a demand arrives, required number of finished products in inventory are released their kanban cards and sent to customer. A kanban card is attached to a container and sent to the beginning workstation to refill the sent products. This kanban card is called production kanban. There is another type of kanban cards named ”two-card system” also known as Toyota Production System. In a ”two-card system”, transportation kanban cards are used in addition to production kanbans. Transportation kanbans give us the information of the quantity of semi-finished products that should be pulled from last workstation to the previous one, also pulled from supplier to manufacturer. Considering the definition, one of these cards is used to trigger production and the other one is used for the movement of products in a ‘two-card system’. The advantage of ‘one-card system’ against two-card system is its ease in application.

Moreover, in this paper simply we dealt with a single-stage manufacturing system because this is the easiest way of distributing the tasks of the system. From this point of

view our single-stage kanban system is same as the constant work-in-process (CONWIP) controlled production system. In CONWIP controlled production system when the production of a container is finished at the last workstation, this last workstation sends an authorized card to the initial workstation that means start to the production of the new container (Tardif & Maaseidvaag 2001). The main difference between a traditional kanban system and a CONWIP system is that in traditional kanban system there are different kanban cards for each component of a product but in CONWIP system there is only one card which is used for all components of a product. According to this a CONWIP system is easier to apply.

On the other hand, traditional kanban control system is applicable for the manufacturing systems that has special conditions such as stable demand rate, constant lead time, fixed processing times, no defects and breakdowns. These situations can be provided in theory but they are not possible in practice. In real life conditions manufacturing companies face with demand uncertainties, variations in processing times and lead times, sudden defects and breakdowns. Manufacturing companies need a new mechanism to handle these problems. Adaptive kanban control approach arised to fill this gap. In this paper, we aimed to provide a formal model for this art of business, and shed light for practitioners by presenting quantitative insights about the benefits of such approach.

1.2 ORGANIZATION OF DISSERTATION

This thesis is organized as follows:

- Chapter 1 includes an introduction to kanban control systems and the objective of this thesis.
- In chapter 2, there is a brief literature review about the idea of traditional kanban control systems to dynamically updated kanban systems.
- In chapter 3, problem descriptions and the models are given. This chapter is divided into to two cases. The models and numerical results are given for both cases.
- In chapter 4, the thesis is concluded with main insights and outcomes gained throughout the thesis work process.

2. LITERATURE REVIEW

The JIT approach to manufacturing control with ‘Kanbans’ has received much attention in the last decade (Huang & Kusiak 1996). Kanban control systems are perfectly applicable to stable manufacturing environments that has constant demand and balanced production times. However, it is hard to integrate traditional kanban control systems to unstable manufacturing environments that has uncertain demand and unbalanced production times. This situation leads to a huge gap between academic literature and manufacturing environments. There are some researches offering a little guidance for filling this gap and integrating the kanban systems into these manufacturing environments.

Moreover Huang & Kusiak (1996), Akturk & Erhun (1999), Kumar & Panneerselvam (2007) and Junior & Filho (2010)’s papers are useful for the detailed literature surveys that are published about the design and classification of the different kanban systems. These studies provide a comprehensive framework for the recent status of the literature in this field. In that respect, Huang & Kusiak (1996)’s paper represents an elaborate overview of Kanban systems. The authors indicated that kanban systems are suitable for recursive and stable manufacturing environments but they are not suitable for unsteady and uncertain manufacturing environments. In this study kanban systems are identified explicitly and categorized as single kanban system, dual kanban system and semi-dual kanban system. Also the research approaches implemented to kanban systems are divided in three categories simulation models, mathematical programming models and stochastic models. According to authors it is important to examine the characteristics of kanban systems and determine the number of kanbans. In this study different kanban systems and comparison of them are considered. From that point, JIT operated companies should have less suppliers compared to non-JIT companies. Also there are some expectations about JIT operated companies such as changing the products rapidly according to changes in demand and to be more responsive in changing the routes of jobs in case of a broken machine. The authors presented some case studies to exemplify the kanban systems. Previous studies about kanban systems are gathered together and they took part in the case studies. As a result of these case studies, Huang & Kusiak (1996) stated that kanban systems is a stock policy in its basic form but many advantages come up when it is integrated with Jidoka, setup reduction and quality control circles. Also decreasing the mean length and waiting time in a kanban system depends on decreasing the lot size.

Furthermore Akturk & Erhun (1999)'s paper present a detailed literature survey and classification of methods to state the design parameters and kanban sequences for JIT systems. Firstly they determined the model structures, solution approach, decision variables, performance measures, objective, setting and the kanban type of a kanban control system under some assumptions. They gathered these informations and generated a table. In this table, they classified the studies on kanban systems according to their design parameters. They indicated that there is a considerable relationship between the number and sizes of kanbans and the scheduling decisions. Secondly, some studies are mentioned about determining the kanban sequences. These refers the effect of operational issues on the design parameters. Also the authors generated an experimental model to measure withdrawal cycle lengths, number, sizes and sequences of kanbans and the effect of operational issues such as sequencing rules and actual lead times. The aim of this experimental model is to minimize the total cost in a multi-item, multi-stage, multi-horizon kanban system. This experimental model is examined under different conditions. There are four different rules taken into consideration to determine the kanban sequence. These are shortest processing time (SPT), SPT-F, first come first served (FCFS) and FCFS-F. According to the examination of experimental models, it is shown that the existing JIT studies don't consider the effect of operational issues on the design parameters. Moreover, it is required to examine the scheduling in kanban systems explicitly under different conditions. In addition to this, it is indicated that it is better to use FCFS rule in a system that has long withdrawal cycle lengths instead of instantaneous one. Akturk & Erhun (1999) examined the effect of operational issues by the use of four kanban sequencing rules.

On the other hand, Kumar & Panneerselvam (2007) represents a detailed literature review of JIT-Kanban system such as Akturk & Erhun (1999). The authors presented the philosophy of JIT concept, push and pull systems. Also blocking mechanism in the kanban systems are defined and divided into three categories such as single card- instantaneous, two card-non instantaneous and blocking mechanism operative on material handling. Moreover the significance of measure of performance (MOP) is mentioned and various performance measure factors that are used by different researchers such as average WIP, demand, average kanban waiting time in queue etc. are shown. Additionally, different modeling approaches are categorized such as mathematical model, queueing model, markovians model, simulation and cost minimization model and are discussed. Besides the traditional kanban system, there are some special cases that are discussed in this study. According to authors, in the developing global production environments it is important for companies to increase their profit based on decreasing their costs. For this

purpose JIT-Kanban systems are very beneficial and commonly used production control systems that minimize work-in-process and throughput time and maximize line efficiency. Kumar & Panneerselvam (2007) states that their paper will be a guide for researchers who are interested in JIT-Kanban control systems.

A major part of the academic literature, both in production control and inventory management, proposes models based on the assumption that control parameters are fixed through time. In this thesis we named these models as static kanban systems. In the context of kanban controlled pull-type manufacturing systems, the number of kanban cards is an important control parameter that affects finished goods inventory and work-in-process (WIP) levels of the manufacturing system. Philipoom et al. (1987) searched for the factors that affect the number of kanbans and examined the effects of these factors examined with simulation method. It is indicated that one of the factors that affects the number of kanbans is throughput velocity. When throughput velocity decreases, there will be a necessity for more kanban cards depending on the increase in backordering probability. The other factor that affects the number of kanbans is the coefficient of variation of the processing time. An increase in the coefficient of variation of the processing time increases the necessity for kanban cards. The third factor that affects the increase in number of kanbans is machine utilization. The last factor that affects the number of kanbans is whether or not there exists autocorrelated processing times. A simulation case is prepared to illustrate if the formulas derived to find the factors affecting the number of kanbans is valid or not under a real production environment that has difficulties. Also this case is used to determine the lead times and the number of Kanbans needed to avoid backordering. This simulation example shows that the formulas and the methodology that are given by Philipoom et al. (1987) to determine the number of kanbans are efficient.

There is a different point of view of Junior & Filho (2010) about various modified kanban systems. In this study, various modified kanban systems are identified and classified as systems that follow kanban original logic and systems do not follow the original kanban logic. As we mentioned before JIT-kanban systems work best under perfect conditions and they are not applicable to real life production environment. Based on the literature reviewed in this paper it is shown that %71.9 of modified kanban systems follow the original kanban functioning logic and only %26 of them are applied in practice. Also %28.1 of modified kanban systems do not follow the original kanban functioning logic and %33.3 of them are applied in practice. The theoretical papers that are worked on present a better performance with unsteady demand and production times. In the studies

applied in practice, some other difficulties are considered besides unsteady demand and production times. According to Junior & Filho (2010), these difficulties are distances between workstations, confusion in the flow of materials and having a lot of suppliers.

Besides the large number of benefits, the common idea of the academic literature is that traditional kanban system is not adequate and appropriate for real productive environments. Therefore a new kanban system logic is emerged to meet the needs of real productive environments. The new kanban control system is called adaptive kanban system. We propose a different approach for adaptive kanban systems and a different technique for updating the number of kanbans. We model the update frequency as a measure of the way that the planner reacts to changing status information. There is a limited number of techniques proposed in the literature for adaptive kanban systems. Our approach to developing an adaptive kanban system is different from the available techniques in the literature such that we explicitly incorporate the concept of update frequency. In our study, we are interested in papers that have the topic of dynamically updated adaptive kanban systems.

The first study that suggests to dynamically update the number of kanbans and generates numerical results belongs to Rees et al. (1987). This study is based on equation 2.1 and suggests to compute and update the lead time in equation 2.1 with statistical methods. Equation 2.1 is used to determine the number of kanbans in the plants of Toyota, the company that presents the Just-In-Time approach to the world. (Monden (1983), Ohno (1988) and Co & Sharafali (1997)):

$$K = \frac{D \cdot L(1 + \alpha)}{Q} \quad (2.1)$$

In this formula K indicates the number of kanbans, D is the amount of demand per unit of time, L is the lead time, α is the safety factor and Q is the container size. According to Kumar & Panneerselvam (2007) waiting time, processing time, conveyance time and kanban collecting time are contained in lead time. The safety factor is used to prevent stockouts against sudden changes in demand and supply. An increase in the number of kanbans, leads to an increase in inventory and as a result a surplus exists. On the other hand a decrease in the number of kanbans, leads to a decrease in inventory so a

shortage occurs. The logic of this basic formula is that the number of kanbans have to be determined to satisfy the demand during the production process.

According to the study of Rees et al. (1987), as a result of periodically updating the lead time parameter, there will be a change in the number of kanbans. It is shown that when a sample production line is modeled by simulation, the number of kanbans that are given wrong at the beginning will be updated quickly to the right value. The method that is used in this study is based on statistical data analysis and data collection so it is hard to implement it in practice.

The study of Chaudhury & Whinston (1990) is a push and pull hybrid production system that is not concerned with the number of kanbans or the parameters of kanban system, it is concerned with the adaptation of stochastic product routes between the workstations. The manner of this system is modeled by stochastic automata methods. The objectives of the control system in this paper are to minimize the average throughput time between workstations, to allocate equal loading to each machine in a workstation and to adjust a constantly modifying behaviour according to changing loading conditions. A simulation model applied to the control system. In this simulation model, jobs are scheduled within five workstation. The arriving jobs are sent to each workstation with a Poisson distribution. The service times at each workstation are exponentially distributed. This simulation model indicates that allocating jobs to workstations with adjusting for changing loading situations is superior to a temporary allocation without any adaptation for changing loading situations. Likewise, Quintana et al. (1997) used Kalman Filters technique to dynamically update work-in-process inventory level while minimizing shipping in multiple stage Kanban system. Kalman Filters technique provides the information of required amount of products for the first workstation to satisfy the desired production rate. According to Quintana et al. (1997), the benefits of using Kalman Filters technique is that it is a minimum variance estimator, it is utilized from all the previous information about the system. Quintana et al. (1997) integrated Kalman Filters technique successfully to their adaptive pull type control process for an assembly line.

Likewise Rees et al. (1987), the research of Kotani (2007) is based on equation 2.1. According to Kotani (2007), Toyota company constitutes a new kanban system named e-Kanban that provides information technologies and communication network between Toyota and their suppliers. The figure 2.1 illustrates the e-Kanban system. In this study, equation 2.1 is used to determine the number of kanbans. The order time for e-kanban system is derived from subtraction of lead time from delivery time to the Toyota factory of

the order. Before changing the number of kanbans, two issues should be addressed. First one is what should be the time between orders and the second one is that what should be the quantity of kanbans. Also there are two important points should be obtained. One of them is to prevent the jump in fluctuation in order quantity. The second one is the number of kanbans should be changed at sufficient order times. The main idea of this study is updating separately the number of kanbans that arise from the flow of normal kanban system and the number of kanbans for safety stock. The difference between the demand for current month and next month is considered while determining the number of kanbans by the kanban cycle. The e-Kanban method implemented successfully to three different cases. In first case it is shown that in a manual kanban system, it is really hard to adjust the number of kanbans if a huge number of kanban cards are needed to add to system. Consequently the orders are put before the desired order time and parts took up warehouse spaces of the assembly line. On the other hand in an e-Kanban system it is easy to adjust the number of kanbans and there is no need for preparing the kanbans. Kotani (2007) denotes that e-Kanban system is preferable and applicable against traditional kanban system.

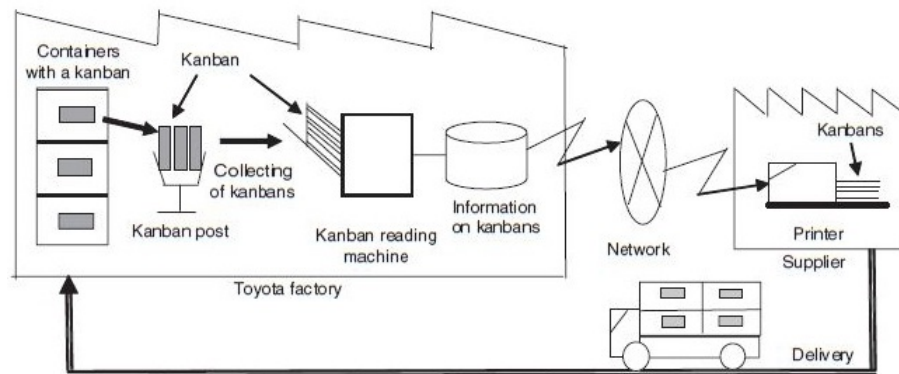


Figure 2.1: An example of e-Kanban system

The idea of dynamically updating the number of kanbans in uncertain and non-stationary conditions also arised from Rees et al. (1987) and adopted by the other researchers. Gupta & Al-Turki (1997) used the term flexible kanban systems (FKS) for the dynamically updated kanban systems. Flexible kanban systems term arised from the need of developing JIT systems that are in use at unbalanced demand and production environment. A new algorithm is designed to manage the number of kanbans in a system with unbalanced demand and production times and to decrease the work-in-process inventory and backorders. The main objective of flexible kanban systems is to increase or decrease the number of kanbans to minimize the starving and blocking due to unbalanced and uncertain production environment. As an example it is needed to increase the number of kanbans to

prevent blocking but the number of kanbans can not exceed the requested amount of demand. There are base level of kanbans in the system at the beginning. The number of kanbans can not be reduced under the base level. Likewise Rees et al. (1987), the number of kanbans are determined upon the lead time and the demand's probabilistic distribution relying on a certain confidence interval. Performance measurement in the system is done by aid of simulation method. According to these encouraging results, flexible kanban systems perform better under unbalanced demand and production conditions. Additionally, Gupta et al. (1999), compared the flexible kanban system with traditional kanban system (TKS) under different uncertain and non-stationary demand conditions using simulation method. In their study they work on four different cases to specify better performance of flexible kanban system against traditional kanban system. These cases are; a JIT system with stochastic production conditions, a JIT system with stochastic production and unstable demand conditions, a JIT system with preventive maintenance under stochastic production and unstable demand conditions and finally a JIT system in a stochastic production and material handling system breakdown conditions. As like as the study of Gupta & Al-Turki (1997), the system's performance is evaluated with simulation method. There are four performance measurements considered in each cases. These are; average time in the system, the average order completion time, average work-in-process and average unit of backorders. The outcome of first case is that FKS decreases the average number of backorders and average order completion time compared to TKS. Second case concludes without any backorders with a little increase of average WIP and average time in system. Third case indicates that FKS gives the optimal backorder and order completion time with a low increase in the average WIP and average time in system. Last case represents that FKS has a low order completion time with a high level of usage of resources. As a result, Gupta et al. (1999) show that flexible kanban system is more productive and effective than the traditional kanban system.

The upper and lower control limits are employed by Takahashi & Nakamura (1999) to update the number of kanbans under non-stationary demand conditions. Two different JIT ordering system is used in this study. These are kanban systems and concurrent ordering system. A Kanban system is triggered by three different datas; arrival of demand, supply of the parts and implementation of the process. On the other hand, concurrent system is triggered by only arrival of demand. Both of these JIT ordering systems are compared with a Material Requirements Planning system (MRP). The simulation results reveals that JIT ordering system accomplish a preferable performance than MRP system because MRP systems utilized from forecasts of demand this may leads to a fallibility. In

the JIT ordering systems when the number of kanbans are stable and they are not sensitive to changes. In the study of Takahashi & Nakamura (1999) a reactive update technique is used to determine the number of kanbans in such a production environment with inequalities in average demand. In addition, with the same update technique Takahashi & Nakamura (2002a) and Takahashi et al. (2004) worked on not only for the inequalities in average demand, at the same time they worked on the inequalities that consists of the disorders on demand variance. Takahashi & Nakamura (2002a) indicates that decentralization is more applicable to JIT systems. Decentralization leads every production stage to control and determine their own number of kanban. Inequalities in demand and disorders in demand variance are found by the use of control charts. In these paper there is a controller for each stage that react according to uncertain changes in demand and determine the number of kanbans. This decentralized kanban system compared to centralized kanban systems states that both of the systems performs at uncertain demand environments. Decentralized kanban system requires less work-in-process (WIP) inventory. Therefore decentralized system displays better performance than centralized system. In these studies, the efficiency of dynamically updated kanban systems is shown by simulations results. Moreover Takahashi & Nakamura (2002b) compared the adaptive kanban systems with the adaptive CONWIP systems. CONWIP systems are pull control systems which are developed as an alternative for kanban systems (Spearman et al. 1990) and it is concerned with fixing semi-finished product level in the manufacturing process. There are two performance measures of this system. These are; average waiting time of demand and average work-in-process (WIP) inventory. It is shown that traditional kanban system has a high level of average WIP inventory compared to reactive kanban systems and reactive CONWIP system has a higher or almost equal level of average WIP inventory compared to traditional CONWIP system. As understood from these results, reactive kanban system is more productive than reactive CONWIP system under uncertain demand conditions. Also Hopp & Roof (1998) developed a system to catch the target production quantity based on the update of semi-finished product level with statistical control methods. In this paper they prefer to utilize from the concept of dynamic step sizes instead of decreasing or increasing the number of cards one by one. Hopp & Roof (1998) demonstrated that the usage of dynamic step sizes minimize the time to catch target production quantity.

Moreover Liu & Huang (2009) worked on a system that is similar to the study of Hopp & Roof (1998). They adjusted a target value of throughput and compared with real time throughput rate. They used the difference between them to accomplish a given throughput

rate. Their system brings out efficient results for card setting in CONWIP system. The number of cards are increased or decreased according to average WIP inventory level. The system replicates until it achieves the target value of throughput. When target value of throughput is achieved, the system continues to work in a constant situation. Also Liu & Huang (2009) researched the impact of processing time on WIP level of system. It is pointed out that distribution of the processing time has a significant impact on WIP level therefore on the target value of throughput rate. Masin & Prabhu (2009) formed feedback control mechanism with simulation method and by this way they aimed to update the number of kanbans at each level and achieve the minimum level of semifinished inventory level at a particular production quantity. Masin & Prabhu (2009) demonstrated that their feedback control mechanism named Adaptive Work-in-Process (AWIP) needs 50 percent less Work-in-Process (WIP) inventory level with respect to Toyota's formula 2.1 used to determine the optimal number of kanbans.

As an alternative for statistical control methods Tardif & Maaseidvaag (2001) came up with another idea that the number of kanbans in the system are changed considering the inventory and backorder levels. Therefore when a demand appears and if inventory level is below a certain level, an extra kanban is released to the system and if it is above a certain level, an extra kanban card is reverted from the system. The system that is modeled as a Markov Chain is solved and compared with a system that has fixed number of kanbans by the aid of simulation. After this study, Takahashi (2003) worked on a study that compares an adaptive system based on statistical control with another adaptive system based on inventory level. Also a new system based on inventory level is presented in this paper and system performances of these three systems are compared with each other. There are two performance measures of this system. These are; average waiting time of demand that measures the customer service level and average Work-in-Process inventory level that measures the productivity of the system. The purpose of these three system is to reduce average WIP inventory level meanwhile keeping the average waiting time of demand as short as possible. All of these systems achieve their purpose. However, this study shows that the adaptive system based on statistical control is more sensitive to the unbalanced changes in demand.

Moreover Takahashi et al. (2010) have another study that the number of kanbans are updated based on inventory level. In this study, they worked on a two stage production system and the problem of updating number of kanbans for each level. Also Shahabudeen & Sivakumar (2008) and Sivakumar & Shahabudeen (2009), worked on an update technique

Authors	Year	System Type		Model Structure			Type of Threshold	
		Single-stage	Multi-stage	Simulation	Markov Chain	Statistical P. C.	Static Threshold	Dynamic Threshold
Gupta et. al	1997		✓	✓		✓	✓	
Hopp et al.	1998	✓		✓		✓	✓	
Gupta et al.	1999		✓	✓		✓	✓	
Takahashi et al.	1999		✓	✓		✓	✓	
Tardif et al.	2001	✓			✓		✓	
Takahashi et al.	2002a	✓	✓	✓		✓	✓	
Takahashi et al.	2002b		✓	✓		✓	✓	
Takahashi et al.	2003		✓	✓		✓	✓	
Takahashi et al.	2004		✓	✓		✓	✓	
Shahabudeen et al.	2008	✓		✓		✓	✓	
Liu et al.	2009		✓	✓		✓	✓	
Masin et al.	2009		✓	✓		✓	✓	
Sivakumar et al.	2009		✓	✓			✓	
Our Study	2013	✓			✓			✓

Table 2.1: Review of kanban systems with different design parameters

based on inventory level. Both of these studies aimed to reduce the total cost through minimizing WIP inventory and backlog. Shahabudeen & Sivakumar (2008) worked on a single stage manufacturing system and Sivakumar & Shahabudeen (2009) improved this system to a multiple stage manufacturing system. Shahabudeen & Sivakumar (2008) utilized from generic algorithm and simulation annealing in their study. The results indicates that adaptive kanban system based on simulation annealing concept perform better results. Sivakumar & Shahabudeen (2009) utilized from generic algorithm in their multiple stage adaptive kanban system and they denoted that the number of kanban cards needed in adaptive kanban system is less than traditional kanban system. Both of these studies presents the ascendancy of adaptive kanban system over traditional kanban system.

Table 2.1 is related to the literature review of adaptive kanban systems with different design parameters. We classified these studies under three important parts. These are system type, model structure and the type of threshold used to update the number of kanban cards that are considered in these studies.

In brief, when we look at table 2.1, we will see that generally the studies are concentrated on statistical control methods and performance measurement is done by aid of simulation method. Stastistical control mechanisms needs a lot of data collection, data storage and data analysis. In real production environments, it is very hard to collect data and it won't be reliable. Same conditions are valid for clarifying and analyzing these datas. It is hard to implement these methods under these conditions. Therefore, there is a need for more basic

and applicable methods. Moreover, these studies are based on static threshold levels. It will be challenging to solve this problem with stochastic modeling approach, alternatively to the simulation method, to reach an exact solution and to provide optimum decisions. Therefore, we utilized from the concept of dynamic threshold levels such that the changes on our system depends on the number of extra kanbans. We modeled our single stage manufacturing system as a continuous time Markov Chain and solved explicitly. We generate important managerial insights on the design of adaptive kanban systems and provide numerical results that compared with static systems where the number of kanbans are kept fixed.

3. PROBLEM DESCRIPTION AND THE MODELS

The simplest way of allocating functions and resources of a manufacturing system to different stages is to consider the entire manufacturing system as a single stage (Tardif & Maaseidvaag 2001). In this case, the kanban control mechanism is equivalent to the CONWIP mechanism. In this thesis, we focused our attention on such single-stage pull control systems and we modeled both static and adaptive kanban systems as a queueing network model. We worked on two different cases. The difference of these two cases is the number of end products produced per unit of time (λ_p). In the first case production rate of λ_p has a fixed rate. In the second case production rate of $\lambda_{p(n)}$ changes according to the WIP level n in the Manufacturing Process (MP). Figure 3.1 below is the queueing network model of static kanban system for case 1:

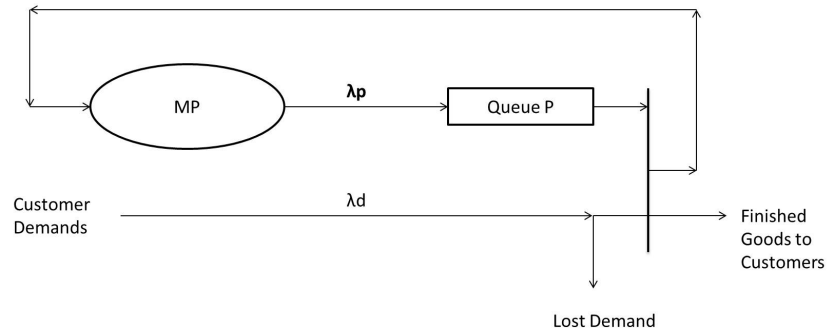


Figure 3.1: Queueing network model for static kanban system for case 1.

In static kanban system for case 1, demand arrives at a manufacturing process (MP) according to a fixed rate λ_d and depletes the finished items in queue P. Kanban cards are attached to items in queue P and in MP. According to kanban logic, each time a demand arrives, the kanban card attached to an item is released and authorizes production in MP. In turn, for case 1 MP finishes production at a fixed rate of λ_p . We assume that back orders are not allowed in the both cases so the size of queue D is always zero in which case there exists lost demand.

Figure 3.2 represents the queueing network model of adaptive kanban system for case 1 and figure 3.3 represents the queueing network model of adaptive kanban system for case 2 where the number of kanban cards vary dynamically according to current inventory level:

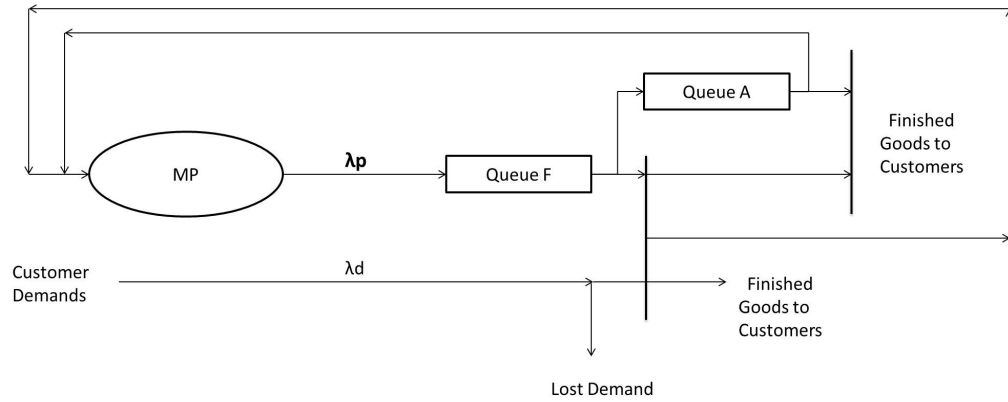


Figure 3.2: Queueing network model for adaptive kanban system for case 1.

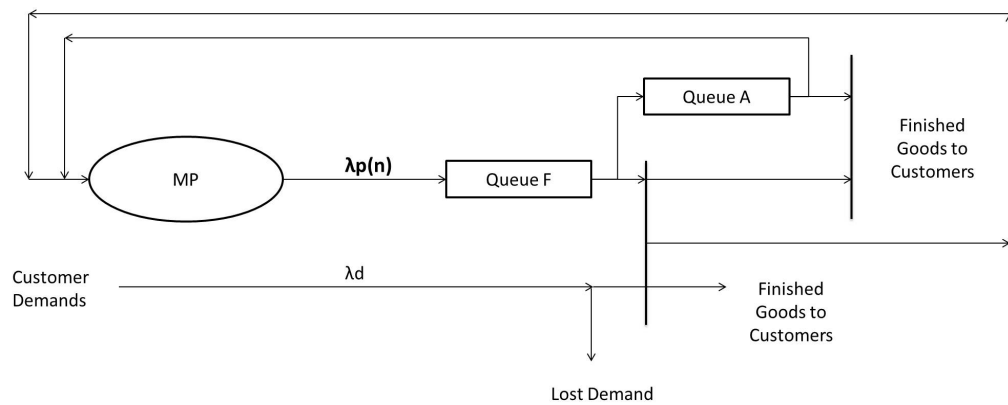


Figure 3.3: Queueing network model for adaptive kanban system for case 2.

The mechanism logic is same for adaptive kanban system in both cases. Again MP is the manufacturing process, queue F contains the finished parts. Queue A contains the extra kanban cards. K is the inventory level at the primal state of the manufacturing system, E is the number of extra kanban cards and r is kanban updating parameter. When a customer demand arrives to the system, if the current inventory level is between $K + E$ and $K - (E + 1) \cdot r$ where the number of extra kanban card in the system is between 0 and E_{max} , a finished part in queue F releases it's kanban card and it is sent to the customer. The released kanban card attached to a demand to produce a new part and it is sent to the MP. If the current inventory level is less than $K - (E + 1) \cdot r$ which means a decrease in on hand inventory by an amount of r where the number of extra kanban card in the system is between 0 and $E_{max} - 1$, an extra kanban card is released to the system from queue A and sent to MP. Then a part releases its card and it is sent to the customer. If the number of finished parts in queue F is greater than $K + E$ which means an increase on on hand inventory by an amount of r where the number of extra kanban card in the system is between 1 to E_{max} , the card releases from a part is not sent to MP. It is sent to extra

kanban cards queue A. If the current inventory level is zero when a demand arrives, lost demand occurs.

Furthermore, it is assumed that arrival of demand per unit of time to the production system is a Poisson process and the number of end products produced per unit of time has an Exponential distribution for both systems. These assumptions are based on the study of Tardif & Maaseidvaag (2001). Under these assumptions, the mathematical models that represent static and adaptive Kanban system are generated as a continuous time Markov Chain for case 1. In this case we analyzed the characteristics of static and our adaptive Kanban system, compared them with each other and determined the optimal number of Kanbans. In the second case, three alternative $K + E_{max}$ values are chosen to evaluate the effect of r on three different service levels such as low, medium and high. Both cases are solved explicitly in the following section.

3.1 CASE 1

3.1.1 The Static Kanban Model

The static kanban control system is represented by a Markov Chain diagram in Figure 3.4. Model parameters and descriptions of this figure 3.4 are as follows; λ_d is the arrival of demand for end product per unit of time, λ_p is the number of end products produced per unit of time $t \geq 0$, K is the number of kanbans at the primal state of the manufacturing system. α is the fraction of the number of end product produced per unit of time $t \geq 0$ to the arrival of demand for end product per unit of time. Also n is equal to the number of Kanbans at the beginning and P_n indicates the probability that there are n number of kanbans at the finished goods inventory. Accordingly, P_0 indicates the probability that there are no kanbans at the finished goods inventory.

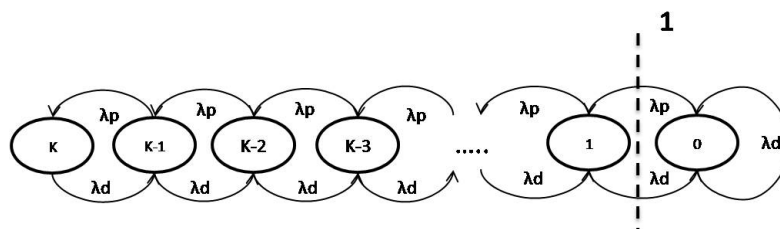


Figure 3.4: Markov Chain model of the Static System for K number of Kanbans for case 1.

In this figure system status is represented by the number of kanbans in the finished goods inventory. When a demand arrives to the system, a kanban card attached to an item is released and triggers the production. The finished goods that are produced in one unit of time is modeled independent of WIP level. The model is solved by Visual Basic program. For the figure 3.4, we calculate the flow balance equation 3.1 for 1:

$$\lambda_d \cdot P_{n+1} = \lambda_p \cdot P_n, \sum_{n=0}^K P_n = 1. \quad (3.1)$$

For $\alpha = \frac{\lambda_p}{\lambda_d}$, we write each steady state probability P_n in terms of P_0 and obtain the following normalization equation 3.2:

$$(\alpha)^0 \cdot P_0 + (\alpha)^1 \cdot P_0 + \dots + (\alpha)^n \cdot P_0 = 1 \quad (3.2)$$

The normalization equation 3.2 can be written as 3.3:

$$P_0 \cdot [1 + (\alpha)^1 + (\alpha)^2 + \dots + (\alpha)^n] = 1 \quad (3.3)$$

According to equation 3.1, we can see that the steady-state probabilities depend on the number of kanbans at the beginning. The steady state probabilities can be written as follows:

$$P_n = (\alpha)^n \cdot P_0 \quad (3.4)$$

Equation 3.5 gives P_0 which indicates the probability that there are no kanbans at the finished goods inventory.

$$P_0 = \frac{1}{\sum_{n=0}^K (\alpha)^n} \quad (3.5)$$

There are three performance criteria of static kanban system:

$E[I]$: Average finished goods inventory level,

$E[WIP]$: Average Work-in-Process inventory level,

$E[L]$: Average Lost Demand per unit time.

Equation 3.6 gives the average finished goods inventory level, equation 3.7 gives the average work-in-process inventory level and equation 3.8 gives the average lost demand:

$$E[I] = \sum_{n=0}^K n \cdot P_n \quad (3.6)$$

$$E[WIP] = 0 \cdot P(n) + 1 \cdot P(n - 1) + 2 \cdot P(n - 2) + \dots + n \cdot P(0) \quad (3.7)$$

$$E[L] = \lambda_d \cdot P_0 \quad (3.8)$$

3.1.2 The Adaptive Kanban Model

In a kanban control system, material flow and production order in the manufacturing system is performed with a certain number of kanban card. In our adaptive kanban system, we keep extra E number of kanban cards in addition to these base level of kanbans. Extra kanban cards are independent of any production order or stock. They are used to update the number of kanbans in the system. According to this we utilized from the concept of update frequency. The update frequency concept that we used in our model, represents the planner's reaction on the changes in inventory levels. A higher update frequency represents a more reactive planner who update the number of kanbans even for small changes in inventory and a lower update frequency represents a less reactive planner who waits till a considerable large change occurs in inventory levels.

We incorporate the concept of update frequency into our model through a fixed parameter r . When the finished goods inventory level decreased by r , an extra kanban card is attached to a production order and released to the system. When the finished goods inventory level increased by r , an extra kanban card is detached from an end product after satisfying the demand and it is sent to extra kanban card queue. In our dynamically updated kanban system, changes depend on the number of extra kanban cards. In other words, the number of active kanban cards in the system have to be less than the summation of number of beginning kanbans and number of extra kanbans and it has to be greater than number of beginning kanbans.

In our system, back orders are not allowed in which case there exists lost demand. When a demand arrives, if the current inventory level is zero, lost demand occurs. Lost demand yields to customer dissatisfaction and this situation leads to a decrease in the customer service level.

Figure 3.5 represents continuous-time Markov Chain for the adaptive kanban system. In this Markov Chain, status information is determined in two dimensions $(N(t), X(t))$. $N(t)$ is the finished goods inventory level at time t , where $K + E \geq N(t) \geq 0$ and $X(t)$ is the number of extra kanban cards that are active in the system at time t , for $E \geq X(t) \geq 0$.

K is the number of finished goods inventory level at the primal state of the manufacturing system, E_{max} is the maximum number of extra kanban cards, r is the kanban updating parameter, λ_d is the arrival of demand for end product per unit of time, λ_p is production rate

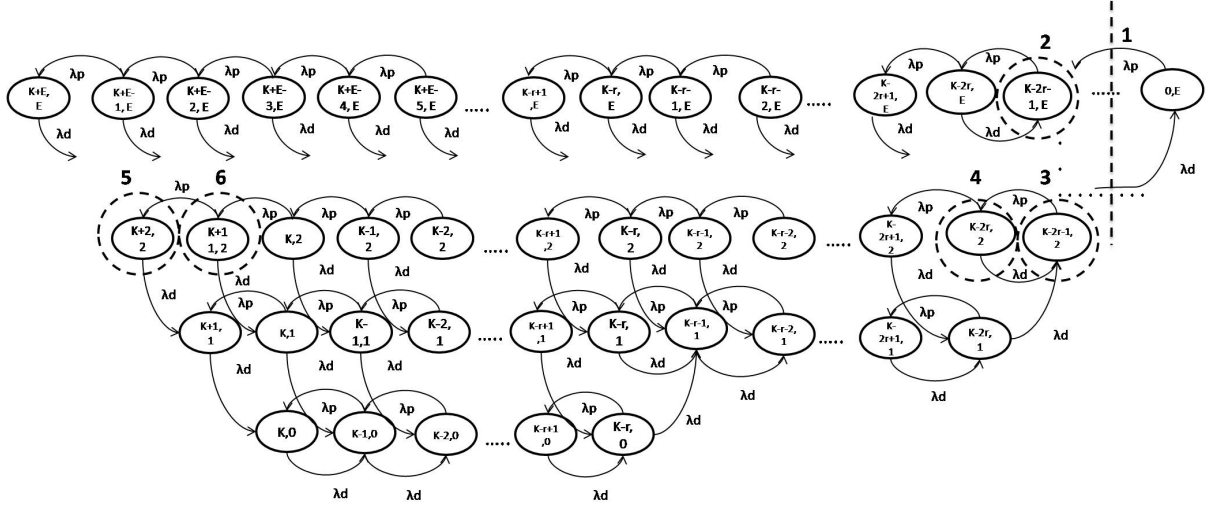


Figure 3.5: Markov Chain diagram representing dynamically updated kanban systems for K, E and r for case 1.

per unit of time $t \geq 0$, $P_{N(t),X(t)}$ is the steady-state probabilities and P_i is the summation of steady-state probabilities for $j = 0, \dots, E_{max}$. P_i can be defined as $P_i = \sum_{j=0}^{E_{max}} P_{i,j}$, for $i = 0, \dots, K + E_{max}$. α is the fraction of the production rate per unit of time $t \geq 0$ to the arrival of demand for end product per unit of time. We solved this continuous-time Markov Chain with the following six flow balance equations:

Flow balance equation for 1:

$$\lambda_d \cdot P_{i+1} = \lambda_p \cdot P_i, \quad \sum_{i=0}^{K+E_{max}} P_i = 1. \quad (3.9)$$

The equation 3.9 can be written as:

$$[(\alpha)^1 + \dots + (\alpha)^{K+E_{max}}] \cdot P_{0,E_{max}} = 1 \quad (3.10)$$

Equations 3.9 and 3.10 give $P_{0,E_{max}}$ which indicates the probability that there are no kanbans at the finished goods inventory.

$$P_{0,E_{max}} = \frac{1}{\sum_{i=1}^{K+E_{max}} (\alpha)^i} \quad (3.11)$$

The summation of steady-state probabilities P_i for $j = 0, \dots, E_{max}$ can be shown as:

For $i= 1, \dots, K + E_{max}$

$$P_i = P_{0,E_{max}} \cdot (\alpha)^i \quad (3.12)$$

The steady state probabilities of the flow balance equation for 1:

For $i= 1, \dots, K - (E_{max} + 1) \cdot r + r - 1$:

$$P_{i,E_{max}} = P_{0,E_{max}} \cdot (\alpha)^i \quad (3.13)$$

Flow balance equation for 2:

For $i= K + E_{max}, \dots, K - (E_{max} + 1) \cdot r + r$ and $h=i-K+(E_{max} + 1) \cdot r - r + 1$

$$P_{i,E_{max}} \cdot [\lambda_p + \lambda_d]^h = P_{(K-(E_{max}+1) \cdot r + r - 1, E_{max})} \cdot (\lambda_p)^h, \quad (3.14)$$

The steady state probabilities of the flow balance equations for 2:

For $i= K - (E_{max} + 1) \cdot r + r, \dots, K + E_{max}$ and $h=i-K+(E_{max} + 1) \cdot r - r + 1$

$$P_{i,E_{max}} = P_{(K-(E_{max}+1)\cdot r+r-1,E_{max})} \cdot \left[\frac{\lambda_p}{\lambda_p + \lambda_d} \right]^h, \quad (3.15)$$

Flow balance equation for 3:

For $j=1,\dots,E_{max} - 1$ and $i=K - (j + 1) \cdot r$

$$P_{i,j} \cdot [\lambda_p + \lambda_d] = [\lambda_d] \cdot P_{i+1} \quad (3.16)$$

The steady state probabilities of the flow balance equations for 3: For $j=1,\dots,E_{max} - 1$ and $i=K - (j + 1) \cdot r$

$$P_{i,j} = \left[\frac{\lambda_d}{\lambda_p + \lambda_d} \right] \cdot P_{i+1} \quad (3.17)$$

Flow balance equation for 4:

For $j=1,\dots,E_{max} - 1$ and $i=K - (j + 1) \cdot r + 1,\dots,K - (j + 1) \cdot r + r - 1$

$$P_{i,j} \cdot [\lambda_p + \lambda_d] = [\lambda_d] \cdot P_{i+1} + [\lambda_p] \cdot P_{i-1,j} \quad (3.18)$$

The steady state probabilities of the flow balance equations for 4:

For $j=1,\dots,E_{max} - 1$ and $i=K - (j + 1) \cdot r + 1,\dots,K - (j + 1) \cdot r + r - 1$

$$P_{i,j} = \left[\frac{\lambda_d}{\lambda_p + \lambda_d} \right] \cdot P_{i+1} + \left[\frac{\lambda_p}{\lambda_p + \lambda_d} \right] \cdot P_{i-1,j} \quad (3.19)$$

The steady state probabilities of the flow balance equations for 5:

For $j=1, \dots, E_{max} - 1$ and $i=K + j$

$$P_{i,j} = P_{i+1,j+1} + (\alpha) \cdot P_{i-1,j} \quad (3.20)$$

Flow balance equation for 6:

For $j=1, \dots, E_{max} - 1$ and $i=K - (j + 1) \cdot r + r, \dots, K + j - 1$

$$P_{i,j} \cdot [\lambda_p + \lambda_d] = [\lambda_d] \cdot P_{i+1,j+1} + [\lambda_p] \cdot P_{i-1,j} \quad (3.21)$$

The steady state probabilities of the flow balance equations for 6:

For $j=1, \dots, E_{max} - 1$ and $i=K - (j + 1) \cdot r + r, \dots, K + j - 1$

$$P_{i,j} = \left[\frac{\lambda_d}{\lambda_p + \lambda_d} \right] \cdot P_{i+1,j+1} + \left[\frac{\lambda_p}{\lambda_p + \lambda_d} \right] \cdot P_{i-1,j} \quad (3.22)$$

Three performance criteria of the adaptive kanban model are defined using these steady-state probabilities:

$E[I]$: Average Finished Goods inventory level,
 $E[WIP]$: Average Work-in-Process inventory level,
 $E[L]$: Average Lost Demand.

At the adaptive Markov chain model, none of the extra kanban cards are active in the system at the beginning and a new extra kanban card is released to the system when finished goods inventory level N_t is equal to $K - (E + 1) * r$, for $E=1, \dots, E_{max}$. In other words, the number of extra kanban cards that are active in the system increase one by one. According to this, equation 3.23 have to be satisfied to complete at least one loop. Therefore, on hand inventory at the beginning depends on two parameters; the number of kanban cards on hand at the initial state of the manufacturing system and the kanban updating parameter.

$$K \geq ((E_{max+1}) \cdot r) - r + 1 \quad (3.23)$$

The service level of the system gives the percentage of the systems performance. When the arrival of demand per unit of time exceeds production rate per unit of time, the service level decreases. On the other hand, when the arrival of demand per unit of time is less than production rate per unit of time, this leads to an increase in the service level. Because the supplementary of service level is running out of stock that means we can not satisfy customer demand. In our system, backorders are not allowed in this situation there exists lost demand. Figure 3.24 shows that summation of the service level and fraction of lost demand per unit of time can not exceed 100 percent which is the full capacity.

$$ServiceLevel \leq 1 - \left(\frac{E[L]}{\lambda_d} \right) \quad (3.24)$$

The equations of the performance criterias; average finished goods inventory level, average work-in-process inventory level, and average lost demand are as follows:

$$E[I] = \sum_{i=0}^{K+E_{max}} i \cdot P_i \quad (3.25)$$

$$E[WIP] = \sum_{n=0}^{E_{max}-1} \sum_{w=n*(r+1)}^{n*(r+1)+r} \sum_{j=n}^{E_{max}} w \cdot P_{K+j-w,j} + \sum_{i=0}^{K-(E_{max}+1)*r+r} (K+E_{max}-i) \cdot P_{i,E_{max}} \quad (3.26)$$

We used the equation for $P_{0,E_{max}}$ which is equal P_0 to define equation 3.27:

$$E[L] = \lambda_d \cdot P_{0,E_{max}} = \frac{\lambda_d}{\sum_{i=1}^{K+E_{max}} \left[\frac{\lambda_p}{\lambda_d} \right]^i} \quad (3.27)$$

Equation (3.28) is obtained from equation (3.24) and (3.27). According to equation (3.28), $K + E_{max}$ value depends on the Service Level, arrival of demand for end product per unit of time and production rate per unit of time.

$$ServiceLevel \leq 1 - \left(\frac{1}{\sum_{i=1}^{K+E_{max}} \left[\frac{\lambda_p}{\lambda_d} \right]^i} \right) \quad (3.28)$$

3.1.3 Numerical Results for Case 1

Our numerical results are based on the formulas given in section 3.1.1 and 3.1.2. We compared the numerical results of adaptive kanban system with the static kanban system. Equation (3.27) demonstrated that average lost demand amount is equal to K in the static system and $K + E_{max}$ in the adaptive system so we equalized K value in the static system to $K + E_{max}$ value in the adaptive system. We found the smallest $K + E_{max}$ value that is acceptable for equation (3.28) and then we used this value to determine K and E_{max} values in the adaptive system according to equation (3.23). A constant average lost demand and a constant service level is determined according to the equation (3.24).

We run our system for different λ_d values such as 7.5, 8, 9, 9.5, 10 and 12 and a fixed λ_p value of 10 and for different service levels such as 0.99, 0.95, 0.9, 0.98, 0.85, 0.80, 0.75, 0.70, 0.65 and 0.60. We obtained the optimal results based on these values. Optimal K , E and r values depend on the summation of average finished goods inventory level and average work-in-process inventory level that minimize the total cost. Therefore our optimal results are K , E and r values that gives the minimum total inventory level for each λ_d , λ_p and service level. Table 3.1 below shows the optimal K , E and r values for our system:

According to the table 3.1, generally the optimal r value that corresponds to update parameter is low at the high service levels. The optimal r values are increasing when the service levels are decreasing. That means utilization decreases corresponding to updating parameter. At the same time utilization increases when update parameter increases. This leads to a better average work-in process level and average finished goods inventory level. Updating the number of kanbans frequently in high service levels leads to better performance and decreases both the average work-in process inventory level and average finished goods inventory level.

Moreover, considering the optimal results, the number of kanbans at the primal state of the manufacturing system is close to the number of extra kanbans we kept on hand at the primal state of the manufacturing system. At a constant production rate, when demand increases the number of kanbans at the primal state of the manufacturing system and the number of extra kanbans kept on hand are increasing to satisfy the demand. When demand

ADAPTIVE SYSTEM							
SERVICE LEVEL	Lambda P	Lambda D	K+E _{max}	r	K	E _{max}	TOTAL INVENTORY
0,99	10	7,5	12	1	7	5	10,254
0,99	10	8	14	1	8	6	11,828
0,99	10	9	23	1	12	11	18,752
0,99	10	9,5	35	1	18	17	28,267
0,99	10	10	100	1	51	49	82,765
0,99	10	12	-	-	-	-	-
0,98	10	7,5	10	1	6	4	8,561
0,98	10	8	11	1	6	5	9,127
0,98	10	9	18	1	10	8	14,902
0,98	10	9,5	25	1	13	12	20,235
0,98	10	10	50	1	26	24	41,465
0,98	10	10	-	-	-	-	-
0,95	10	7,5	7	1	4	3	5,882
0,95	10	8	8	1	5	3	6,902
0,95	10	9	11	1	6	5	9,035
0,95	10	9,5	14	1	8	6	11,764
0,95	10	10	20	1	11	9	16,8
0,95	10	12	-	-	-	-	-
0,9	10	7,5	5	1	3	2	4,261
0,9	10	8	5	1	3	2	4,258
0,9	10	9	7	1	4	3	5,858
0,9	10	9,5	8	1	5	3	6,979
0,9	10	10	10	1	6	4	8,656
0,9	10	12	-	-	-	-	-
0,85	10	7,5	4	2	3	1	3,543
0,85	10	8	4	2	3	1	3,545
0,85	10	9	5	1	3	2	4,27
0,85	10	9,5	6	1	4	2	5,371
0,85	10	10	7	1	4	3	5,91
0,85	10	12	-	-	-	-	-
0,8	10	7,5	3	1	2	1	2,658
0,8	10	8	4	2	3	1	3,545
0,8	10	9	4	2	3	1	3,556
0,8	10	9,5	5	1	3	2	4,284
0,8	10	10	5	1	3	2	4,302
0,8	10	12	64	6	55	9	62,999
0,75	10	7,5	3	1	2	1	2,658
0,75	10	8	3	1	2	1	2,662
0,75	10	9	4	2	3	1	3,556
0,75	10	9,5	4	2	3	1	3,564
0,75	10	10	4	2	3	1	3,575
0,75	10	12	9	1	5	4	7,854
0,7	10	7,5	3	1	2	1	2,658
0,7	10	8	3	1	2	1	2,662
0,7	10	9	3	1	2	1	2,673
0,7	10	9,5	4	2	3	1	3,564
0,7	10	10	4	2	3	1	3,575
0,7	10	12	7	1	4	3	6,1
0,65	10	7,5	2	-	-	-	-
0,65	10	8	3	1	2	1	2,662
0,65	10	9	3	1	2	1	2,673
0,65	10	9,5	3	1	2	1	2,679
0,65	10	10	3	1	2	1	2,687
0,65	10	12	5	1	3	2	4,394
0,6	10	7,5	2	-	-	-	-
0,6	10	8	2	-	-	-	-
0,6	10	9	3	1	2	1	2,673
0,6	10	9,5	3	1	2	1	2,679
0,6	10	10	3	1	2	1	2,687
0,6	10	12	4	2	3	1	3,623

STATIC SYSTEM				
SERVICE LEVEL	Lambda P	Lambda D	K	TOTAL INVENTORY
0,99	10	7,5	12	12
0,99	10	8	14	14
0,99	10	9	23	23
0,99	10	9,5	35	35
0,99	10	10	100	100
0,99	10	12	-	-
0,98	10	7,5	10	10
0,98	10	8	11	11
0,98	10	9	18	18
0,98	10	9,5	25	25
0,98	10	10	50	50
0,98	10	12	-	-
0,95	10	7,5	7	7
0,95	10	8	8	8
0,95	10	9	11	11
0,95	10	9,5	14	14
0,95	10	10	20	20
0,95	10	12	-	-
0,9	10	7,5	5	5
0,9	10	8	5	5
0,9	10	9	7	7
0,9	10	9,5	8	8
0,9	10	10	10	10
0,9	10	12	-	-
0,85	10	7,5	4	4
0,85	10	8	4	4
0,85	10	9	5	5
0,85	10	9,5	6	6
0,85	10	10	7	7
0,85	10	12	-	-
0,8	10	7,5	3	3
0,8	10	8	4	4
0,8	10	9	4	4
0,8	10	9,5	5	5
0,8	10	10	5	5
0,8	10	12	64	64
0,75	10	7,5	3	3
0,75	10	8	3	3
0,75	10	9	4	4
0,75	10	9,5	4	4
0,75	10	10	4	4
0,75	10	12	9	9
0,7	10	7,5	3	3
0,7	10	8	3	3
0,7	10	9	3	3
0,7	10	9,5	4	4
0,7	10	10	4	4
0,7	10	12	7	7
0,65	10	7,5	2	2
0,65	10	8	3	3
0,65	10	9	3	3
0,65	10	9,5	3	3
0,65	10	10	3	3
0,65	10	12	5	5
0,6	10	7,5	2	2
0,6	10	8	2	2
0,6	10	9	3	3
0,6	10	9,5	3	3
0,6	10	10	3	3
0,6	10	12	4	4

Table 3.1: Optimal Results for case 1

rate per unit of time exceeds the production rate per unit of time, both of the systems will not work on high service levels because the system could not satisfy the demand and assure a high service level. In this situation manufacturing system can only provide a low service level.

We consider two different service levels 0.99 and 0.85 as examples and examined explicitly. Example 1: Figure (3.6) shows the changes in inventory levels via different λ_d values under the service level of 0.99.

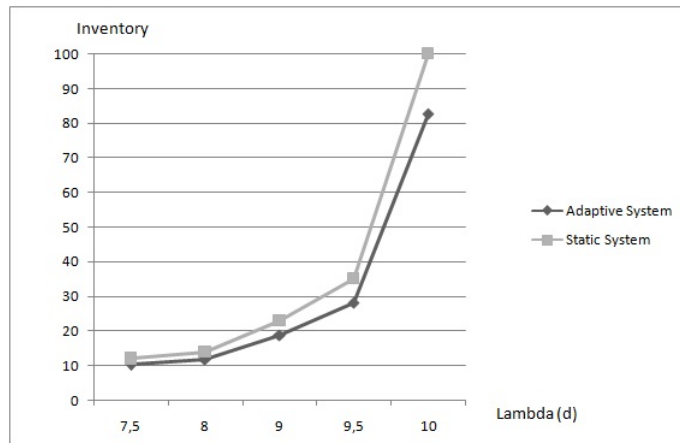


Figure 3.6: Adaptive and Static Systems Inventory Level under the Service Level of 0.99

We considered that both systems has an equal service level of 0.99, an equal lost demand level and a fixed value of 10 for the number of end product produced per unit of time $t \geq 0$. Under these considerations, we run the system for different K , E_{max} and r values based on our formulations to find the optimal K , E_{max} and r . We saw that the optimal value of r is equal to 1 for all λ_d values. That means the update frequency of the system is high so planner should be more reactive to update the number of kanbans for even a small change. Higher update frequency is an advantage for this system.

Example 2: Figure 3.7 shows the changes in inventory levels via different λ_d values under the service level of 0.85. We considered both systems has an equal service level of 0.85, an equal lost demand level and a fixed value of 10 for the number of end product produced per unit of time, $t \geq 0$. In our figure 3.7, we draw three conclusions for the optimal value of r . Firstly, when the arrival of demand per unit of time is low compared to the number of end product produced per unit of time, the number of kanbans updated less frequently. Because the number of kanbans in the system are enough to obtain the desired service level. However when the arrival of demand per unit of time is very close or equal to the

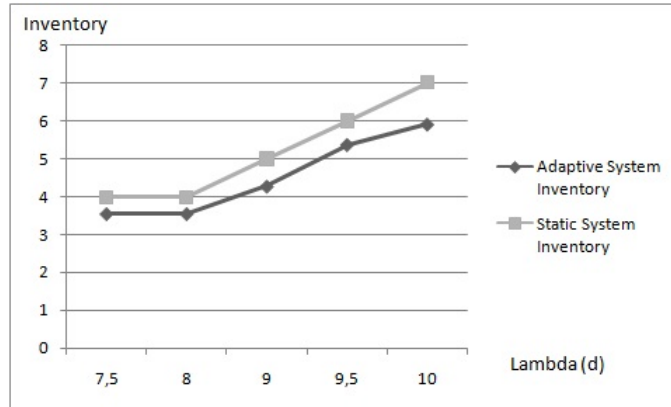


Figure 3.7: Adaptive and Static Systems Inventory Level under the Service Level of 0.85

number of end product produced per unit of time, the number of kanbans updated more frequently. In this situation, an increase in production level to provide desired service level that's the reason of high updating frequency. Thirdly, when the arrival of demand per unit of time exceeds the number of end product produced per unit of time, the system can not perform at 0.85 service level so there is no results for this situation. When the value for arrival of demand per unit of time is 7.5 and 8, the inventory levels seem to be constant because the total inventory level of both systems are almost equal.

Eventually, the total inventory level of adaptive system is less than the static system for all combinations of optimal K , E , r values. With reference to this we conclude that using the update frequency concept is beneficial to decrease the total inventory level. As you see from Figure 3.1, optimal value for $K = 7$, $E = 5$ and $r = 1$ for $\lambda_p = 10$, $\lambda_d = 7.5$ and service level is equal to 0.99. This combination provides a %14.55 improvement over static kanban system. Moreover, Table 3.2 specifies the improvement of adaptive system over static kanban system by percentage for both 0.99 and 0.85 service levels. These improvements are also based on optimal values of K , E and r . When we compare the service levels 0.85 and 0.99, we see that there is a visible difference between the improvements of two service levels. Adaptive model is more effective on higher service level because higher service level means higher utilization. The utilization increases corresponding to the service level. Table 3.2 obviously indicates that adaptive system can achieve crucial improvements in decreasing the total inventory level.

Service Level 0.99					
Lambda D	7,5	8	9	9,5	10
Improvement	14,55%	15,52%	19,48%	19,24%	17,23%
Service Level 0.85					
Lambda D	7,5	8	9	9,5	10
Improvement	11,42%	11,37%	14,60%	10,49%	15,58%

Table 3.2: Improvement of adaptive system over static system for optimal K, E and r values

3.2 CASE 2

3.2.1 The Adaptive Kanban Model

Figure 3.8 represents continuous-time Markov Chain for the adaptive kanban system. In this Markov Chain, status information determined in two dimensions $(N(t), X(t))$. $N(t)$ is the finished goods inventory level at time t , for $K + E \geq N(t) \geq 0$ and $X(t)$ is the number of extra kanban cards that are active in the system at time t , for $E \geq X(t) \geq 0$.

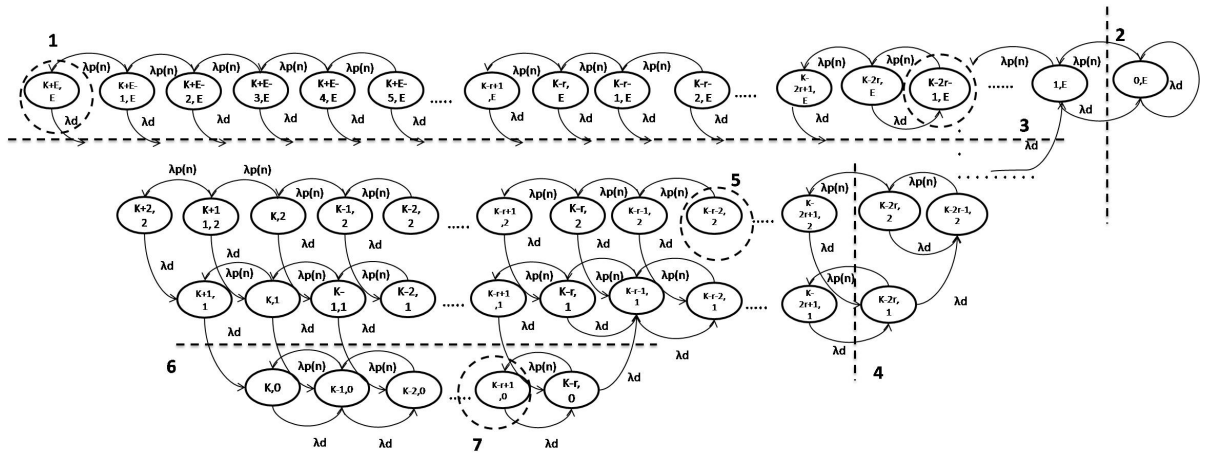


Figure 3.8: Markov Chain diagram representing dynamically updated kanban systems for K, E and r for case 2.

K is the number of finished goods inventory level at the primal state of the manufacturing system, E_{max} is the number of extra kanban cards, r is the kanban updating parameter, λ_d is the arrival of demand for end product per unit of time, $\lambda_p(n)$ is production rate per unit of time $t \geq 0$ where n that varies according to the work-in-process level n in MP, $P_{N(t), X(t)}$ is the steady-state probabilities and P_a is the summation of steady-state probabilities for

$j = 0, \dots, E_{max}$. The processing rate $\lambda_{p(n)}$ is calculated with equation 3.29 in a single stage manufacturing system with b identical parallel servers where μ is the mean rate (Tardif & Maaseidvaag 2001).

$$\lambda_{p(n)} = n \cdot \mu, \quad n \leq b \quad (3.29)$$

We solved this continuous-time Markov Chain with the following seven flow balance equations:

Flow balance equation for 1:

For $j=E_{max}$, $i=K - (j \cdot r) + 1, \dots, K + j$ and $n=K + j - i$

$$P_{i,j+1} \cdot \lambda_{p(n+1)} = P_{i+1,j+1} \cdot (\lambda_{p(n)} + \lambda_d) \quad (3.30)$$

The steady state probabilities of the flow balance equations for 1:

$$P_{i,j+1} = \frac{P_{i+1,j+1} \cdot (\lambda_{p(n)} + \lambda_d)}{\lambda_{p(n+1)}} \quad (3.31)$$

Flow balance equation for 2:

For $j=E_{max}$, $i=0, \dots, K - (j \cdot r) - 1$ and $n=K + j - i$

$$P_{i+1,j+1} \cdot \lambda_{p(n)} = P_{i+2,j+1} \cdot \lambda_d \quad (3.32)$$

The steady state probabilities of the flow balance equations for 2:

$$P_{i+1,j+1} = \frac{P_{i+2,j+1} \cdot \lambda_d}{\lambda_{p(n)}} \quad (3.33)$$

Flow balance equation for 3:

For $j=E_{max}$ and $i=K - (j \cdot r) + 1, \dots, K + j$

$$P_{K-(j \cdot r)+1,j} = P_{i+1,j+1} \quad (3.34)$$

Flow balance equation for 4:

For $j=1, \dots, E_{max} - 1$, $i=K - ((j + 1) * r) + 1, \dots, K - ((j + 1) * r) + r - 1$ and $n=K + j - i$

$$P_{i+1,j+1} \cdot \lambda_d = P_{i,j+2} \cdot \lambda_{p(n+2)} + P_{i,j+1} \cdot \lambda_{p(n+1)} - P_{i+1,j+2} \cdot \lambda_d \quad (3.35)$$

The steady state probabilities of the flow balance equations for 4:

$$P_{i+1,j+1} = \frac{P_{i,j+2} \cdot \lambda_{p(n+2)} + P_{i,j+1} \cdot \lambda_{p(n+1)} - P_{i+1,j+2} \cdot \lambda_d}{\lambda_d} \quad (3.36)$$

Flow balance equation for 5:

For $j=1, \dots, E_{max} - 1$, $i=K - (j + 1) \cdot r + r, \dots, K + j$ and $n=K + j - i$

$$P_{i+1,j+1} \cdot (\lambda_d + \lambda_{p(n)}) = P_{i,j+1} \cdot \lambda_{p(n+1)} + P_{i+2,j+2} \cdot \lambda_d \quad (3.37)$$

The steady state probabilities of the flow balance equations for 5:

$$P_{i+1,j+1} = \frac{P_{i,j+1} \cdot \lambda_{p(n+1)} + P_{i+2,j+2} \cdot \lambda_d}{(\lambda_d + \lambda_{p(n)})} \quad (3.38)$$

Flow balance equation for 6:

For $j=1, \dots, E_{max} - 1$ and $i=K - (j+1) \cdot r + r + 1, \dots, K + j$

$$P_{K-(j \cdot r)+1,j} = P_{i+1,j+1} \quad (3.39)$$

Flow balance equation for 7:

For $j=0, i=K - r + 1, \dots, K$ and $n=K + j - i$

$$P_{i+1,j+1} \cdot (\lambda_d + \lambda_{p(n)}) = P_{i,j+1} \cdot \lambda_{p(n+1)} + P_{i+2,j+1} \cdot \lambda_d + P_{i+2,j+2} \cdot \lambda_d \quad (3.40)$$

The steady state probabilities of the flow balance equations for 7:

$$P_{i+1,j+1} = \frac{P_{i,j+1} \cdot \lambda_{p(n+1)} + P_{i+2,j+1} \cdot \lambda_d + P_{i+2,j+2} \cdot \lambda_d}{(\lambda_d + \lambda_{p(n)})} \quad (3.41)$$

Three performance criterias of the adaptive kanban model are defined using these steady-state probabilities:

$E[I]$: Average finished goods inventory level,

$E[WIP]$: Average Work-in-Process inventory level,

$E[L]$: Average Lost Demand.

There are some assumptions that we defined in case 1. They are valid for case 2 too. As we mentioned before, at the adaptive Markov chain model, none of the extra kanban cards

are active in the system at the beginning and a new extra kanban card is released to the system when finished goods inventory level N_t is equal to $K - (E_{max} + 1) * r$. In other words, the number of extra kanban cards that are active in the system increase one by one. According to this, equation 3.42 have to be satisfied to complete at least one loop. Therefore, on hand inventory at the beginning depends on two parameters; the number of kanban cards on hand at the initial state of the manufacturing system and the update frequency.

$$K \geq ((E_{max+1}) \cdot r) - r + 1 \quad (3.42)$$

The service level of the system gives the percentage of the systems performance. When the arrival of demand per unit of time exceeds production rate per unit of time, the service level decreases. Because the supplementary of service level is running out of stock that means we can not satisfy customer demand. In our system, backorders are not allowed in this situation there exists lost demand. Figure 3.43 shows that summation of the service level and fraction of lost demand per unit of time can not exceed 100 percent which is the full capacity.

$$ServiceLevel \leq 1 - \left(\frac{E[L]}{\lambda_d} \right) \quad (3.43)$$

The equations of the performance criterias; average finished goods inventory level, average work-in-process inventory level, and average lost demand are as follows:

$$E[I] = \sum_{n=0}^{E_{max}-1} \sum_{w=n*(r+1)}^{n*(r+1)+r} \sum_{j=n}^{E_{max}} (K+j-w) \cdot P_{K+j-w,j} + \sum_{i=0}^{K-(E_{max}+1)*r+r} i \cdot P_{i,E_{max}} \quad (3.44)$$

$$E[WIP] = \sum_{n=0}^{E_{max}-1} \sum_{w=n*(r+1)}^{n*(r+1)+r} \sum_{j=n}^{E_{max}} w \cdot P_{K+j-w,j} + \sum_{i=0}^{K-(E_{max}+1)*r+r} (K + E_{max} - i) \cdot P_{i,E_{max}} \quad (3.45)$$

$$E[L] = \lambda_d \cdot P_{0,E_{max}} \quad (3.46)$$

Equation 3.47 is obtained from equation (3.43) and (3.46). According to equation 3.47, the Service Level depends on the probability that there are no kanbans at the finished goods inventory.

$$ServiceLevel \leq 1 - P_{0,E_{max}} \quad (3.47)$$

3.2.2 Numerical Results for Case 2

Our numerical results are based on the formulas given in section 3.2.1. In this section, our aim is to evaluate the effect of r on three different service levels such as low, medium and high. Low service level is nearly %60, medium is %80 and high service level is %99. Therefore, we chose three alternative $K + E_{max}$ values and used these values to determine appropriate K , E_{max} and r values according to equation (3.42). Also we utilized from equation (3.47) to assure the selected service levels. $K + E_{max}$ get the value of 10, 15 and 30 for the service levels low, medium and high respectively.

ADAPTIVE SYSTEM						
SERVICE LEVEL	Lambda D	K+Emax	r	K	Emax	TOTAL INVENTORY
0,59	15	10	1	9	1	9,99
0,59	15	10	2	9	1	9,99
0,59	15	10	3	9	1	9,998
0,59	15	10	4	9	1	9,993
0,59	15	10	5	9	1	9,981
0,58	15	10	6	9	1	9,952
0,57	15	10	7	9	1	9,892
0,53	15	10	8	9	1	9,779
0,82	15	15	1	14	1	15,00
0,82	15	15	2	14	1	15,00
0,82	15	15	3	14	1	15,00
0,82	15	15	4	14	1	15,00
0,82	15	15	5	14	1	15,00
0,82	15	15	6	14	1	14,99
0,82	15	15	7	14	1	14,98
0,81	15	15	8	14	1	14,96
0,81	15	15	9	14	1	14,93
0,80	15	15	10	14	1	14,89
0,78	15	15	11	14	1	14,83
0,74	15	15	12	14	1	14,75
0,68	15	15	13	14	1	14,64
0,99	15	30	1	28	2	29,999
0,99	15	30	2	28	2	29,997
0,99	15	30	3	28	2	29,983
0,99	15	30	4	28	2	29,936
0,99	15	30	5	28	2	29,833
0,99	15	30	6	28	2	29,669
0,99	15	30	7	28	2	29,472
0,99	15	30	8	28	2	29,282
0,99	15	30	9	28	2	29,127
0,99	15	30	10	28	2	29,018
0,99	15	30	11	28	2	28,944
0,99	15	30	12	28	2	28,892
0,99	15	30	13	28	2	28,85

Table 3.3: Change in total inventory level in return for the selected service levels

Furthermore, we determined $\lambda_{p(n)}$ values according to equation (3.29). In our system, we assumed that our system has infinite number of parallel servers and our mean rate value μ is equal to 1 and n is from 1 to $K + E_{max}$ thus, $\lambda_{p(n)}$ get values between 1 and $K + E_{max}$. We run our system for a constant λ_d value 15. The results are obtained based on these values. While enhancing the value of update parameter, r one by one under constant K , E_{max} and λ_d values, we analyze the variation on the total inventory level which is the summation of average finished goods inventory level and average work-in-process inventory level. Table 3.3 below shows the change in total inventory level in return for the selected service levels:

As it is shown in table 3.3, we focused on three different service levels low, medium and high and examined explicitly. These are nearly 0.60, 0.80 and 0.99. We examined the effect of r on these selected service levels.

Figure (3.9) shows the changes in inventory levels via increasing value of r under the low service level. According to figure (3.9), low update parameter leads to a high finished goods and WIP inventory level. While update parameter is increasing finished goods and WIP inventory level is decreasing. On the other hand, manufacturing system provides a lower service level while finished goods and WIP inventory level is decreasing. When we compare the lowest update parameter with highest update parameter for the low service level of 0.60, the improvement of finished goods and WIP inventory level is %2, 11.

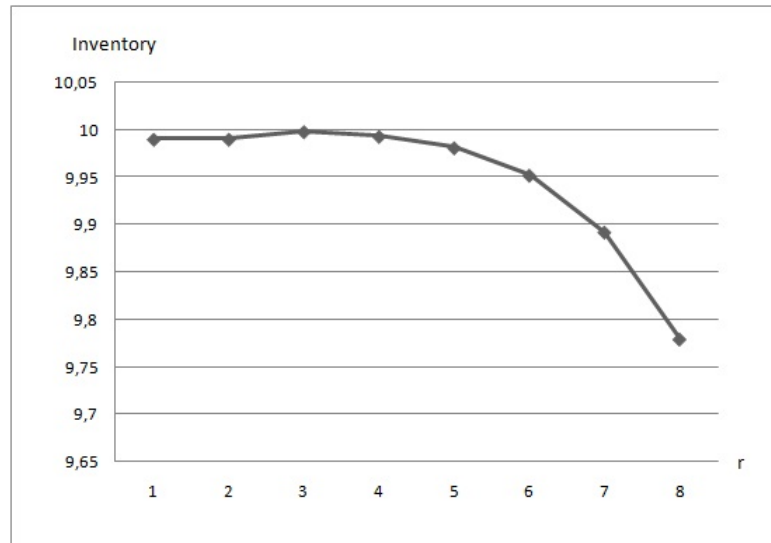


Figure 3.9: Changes in inventory levels via increasing value of r under low service level

Moreover the following figures (3.10) and (3.11) yielded the same results. Likewise when we compare the lowest update parameter with highest update parameter for the medium service level of 0.80, the improvement of finished goods and WIP inventory level is %2, 4. The improvement of finished goods and WIP inventory level for the high service level 0.99 is %3, 831.

Considering these improvements, we could conclude that a lower update frequency is preferable and advantageous for all service levels. This situation presents a less reactive planner who waits till a considerable large change occurs in inventory levels. Update frequency concept can be used as a beneficial control parameter to decrease the finished goods inventory and work-in process inventory levels.

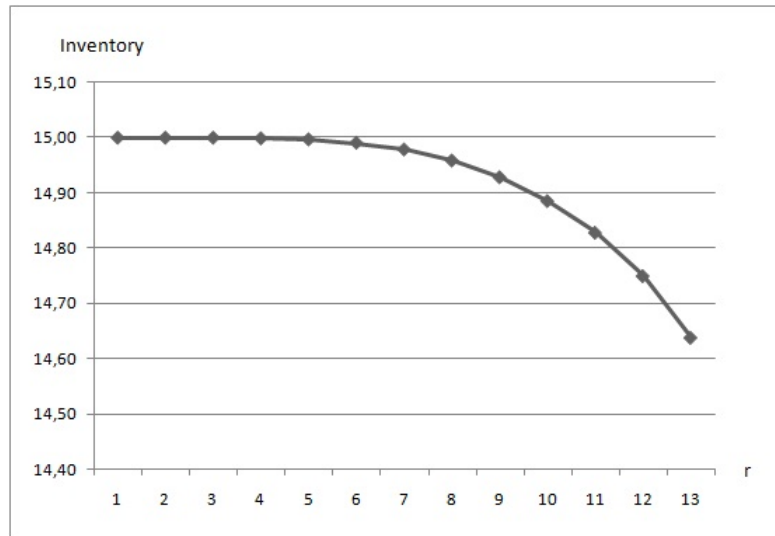


Figure 3.10: Changes in inventory levels via increasing value of r under medium service level

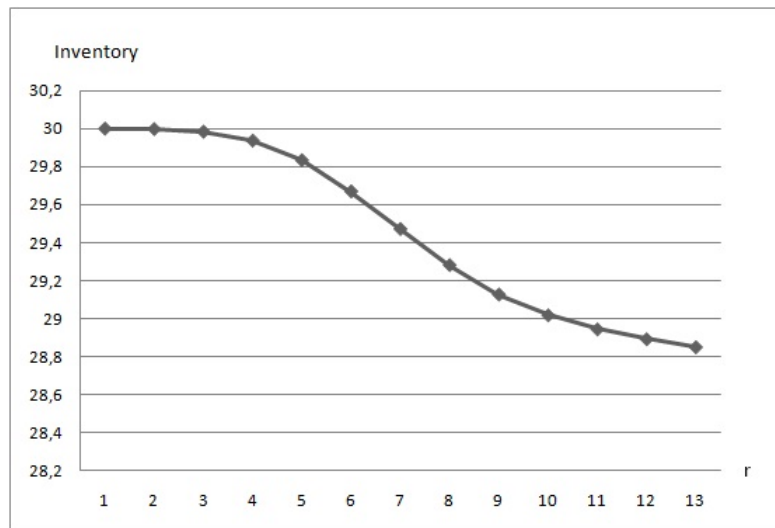


Figure 3.11: Changes in inventory levels via increasing value of r under high service level

4. CONCLUSION

In case 1, we analyzed the characteristics of traditional and our adaptive Kanban system, compared them with each other and determined the optimal number of Kanbans. As you see from the numerical results our adaptive kanban system is superior to the traditional static kanban system under a prescribed set of conditions. The adaptive system is an effective method for controlling and observing the number of kanbans with the use of update frequency concept. The update frequency represents the planner's reaction on the changes in inventory levels. At a higher update frequency, planner must update the number of kanbans even for small changes in inventory. Besides a lower update frequency represents a less reactive planner who waits till a considerable large change occurs in inventory levels.

This paper will be beneficial for stable production conditions with dynamically updated number of kanbans. Important managerial insights are generated for the optimal number of kanbans, optimal number of extra kanbans, update frequency and service level. The main insights generated from numerical results of case 1 are:

1. Utilizing from dynamically updating frequency concept in kanban systems is better than the static kanban systems. The performance of dynamically updated kanban system is higher than static kanban system in given service levels. Dynamically updating the number of kanbans performs much better than the static number of kanbans. The relative improvement is higher when the production system operates under a higher utilization level.
2. The system works under a high service level, if the number of end product produced per unit of time is higher than arrival of demand for end product per unit of time.
3. Under a high service level, in case the number of end product produced per unit of time is higher than arrival of demand for end product per unit of time, higher updating frequency leads to low finished goods inventory and work-in process inventory levels.
4. The system works under a low service level, if the number of end product produced per unit of time is lower than arrival of demand for end product per unit of time.

5. Under a low service level, in case the number of end product produced per unit of time is lower than arrival of demand for end product per unit of time, higher updating frequency leads to low finished goods inventory and work-in process inventory levels.
6. Update frequency concept can be used as a beneficial control parameter to decrease the finished goods inventory and work-in process inventory levels and increase the performance of the system.
7. Implementation of update frequency concept increases system efficiency with decreasing the total inventory level.
8. The adaptive system minimized the total inventory level while simplifying the inventory control.

In case 2, three alternative $K + E_{max}$ values such as 10, 15 and 30 are chosen to evaluate the effect of r on three different service levels such as low, medium and high. The difference of case 1 and 2 is the number of end products produced per unit of time. For case 2, $\lambda_{p(n)}$ varies according to the WIP level n in the MP. The conclusions drawn from numerical results of case 2:

1. When update frequency is high, finished goods and WIP inventory levels is high for our system.
2. Finished goods and WIP inventory level that refers to total inventory level is decreasing, while update frequency is lowering evenly.
3. The improvement of total inventory level is %2.11 for the low service level of 0.60, %2.4 for the medium service level of 0.80 and %3.831 for the high service level of 0.99.
4. The lowest feasible is the optimal update frequency level for this system for each service level.

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