T.C.
BAHÇEŞEHİR ÜNİVERSİTESİ

SOLVING AIRCRAFT ROUTING PROBLEM WITH INTEGER PROGRAMMING

Master's Thesis

Nahit KİRAZOĞLU

Istanbul, 2009

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Nahit Kirazoğlu
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Abstract

SOLVING THE AIRCRAFT ROUTING PROBLEM VIA INTEGER PROGRAMMING
Kirazoğlu, Nahit
Industrial Engineering
Asist. Prof. F. Tunç Bozbura
11, 2009

In airline industry; the nature of competition made it necessary to find new ways for maximizing profits while decreasing operational costs of regular activities. In an airline company there are critical activities such as crew planning, rostering and pairing, aircraft routing, tail assignment and gate assignment. As being one of those most significant ones; this study will handle aircraft routing.

In the first part of this paper, aircraft routing problem is going to be introduced in different dimensions which are objective, several criteria for optimization, constraints encountered during the problem. In the second section, a literature review is completed for making the paper updated due to recent studies. Then in the third part, in order to demonstrate the problem on a real life case data collection method and data collected are reflected. It is important to realize that data are collected from flag carrier of Turkey to simulate the real life. Moreover, data collection method for solving the problem which has been modelled with several assumptions is told.

In the section 4; by using optimization software, Lingo, results are gathered and comments are made on those results which are going to make sense for reader in daily airline operations manner.

In the fifth part; some alternative optimization methods are discussed. Additionally for future studies, a roadmap is designed which includes less assumptions, new constraints and new objective functions meeting needs of changing airline industry elements and environment.

At last, a conclusion will sum up all the sections for the reader and draw a simple understanding about the study.

Keywords: aircraft routing, daily airline operations, flight coverage

Özet

UÇAK ROTALAMA PROBLEMİNİN TAMSAYI PROGRAMLAMA İLE ÇÖZÜLMESİ

Kirazoğlu, Nahit
Endüstri Mühendisliği
Yar. Doç. Dr. F. Tunç Bozbura

11,2009

Havayolu endüstrisinde, rekabetin doğası şirketlere karlarını maksimize etmede ve günlük aktivitelerden doğan operasyonel maliyetlerin azaltılmasında yeni yollar bulma yetisi getirdi. Bir havayolu şirketinde bu yönde aralarında ekip planlama, çiftleme, uçak rotalama, kuyruk atama, kapı atama gibi kritik olanların da bulunduğu çeşitli adımlar bulunmaktadır. Bu bağlamda uçak rotalama bu adımların göz önünde bulundurulması gerekli en önemlilerinden biridir ve bu tezde incelenecektir.

Birinci bölümde, uçak rotalama problemi amacı, en iyileme için değişik kriterleri ve karşılaşılan kısıtlar gibi değişik boyutları ile tanımlanacaktır.

İkinci bölümde ise çalışmayı güncel tutmak adına yakın zamanda konu üzerinde yapılan çalışmaların incelendiği literatür araştırması tamamlanacaktır.

Ardından üçüncü bölümde problemi gerçek hayat vakası olarak gösterebilmek adına veri toplama yöntemi ve toplanan veriler yansıtılacaktır. Verilerin Türkiye’nin bayrak taşıyıcı havayolu şirketinden alınmış olması gerçek hayatı simule etme adına önemlidir. Veri toplamadan sonra bazı varsayımlar üzerine kurulmuş problemi çözme yöntemi anlatılacaktır.

Dördüncü bölümde eniyileme yazılımı kullanılarak sonuçlar elde edilecek ve okuyucunun günlük havayolu operasyonları bağlamında değerlendirebilmesi için bu yönde yorumlar yapılacaktır.

Beşinci bölümde alternatif eniyileme metodları tartışılacaktır. Ek olarak değişen havayolu endüstrisindeki ihtiyaçları karşılamak adına yapılacak ilerideki çalışmalar için daha az varsayımlar, yeni kısıtlar ve yeni hedef fonksiyonlar içeren yol haritası belirlenecektir.

Son olarak, sonuç kısmı bütün bölümleri derleyerek, okur için bütün tezi anlaşılır kılacak.

Anahtar Kelimeler : uçak rotalama, günlük havayolu operasyonları , uçuş kapsama

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LIST OF ABBREVIATIONS

Genetic Algorithm : GA
Aircraft Routing Problem : ARP
Air Transport Association of America : ATA
Federal Aviation Administration : FAA
Integer Linear Programming : ILP

1. INTRODUCTION

1.1. Overall Look on Airline Industry:

There are not so many inventions that effect how people live as much as the invention of airplane. In development of airplane industry, World Wars I and II were milestones for the industry since governments had demanded vast amount of airplanes. In order to meet demands, airplane industry demonstrated a rapid development which created a basis for post-war non-military aviation industry. Day by day, aviation had taken more place in people's lives as it provided short travel times for long distances people used to call remote. Not only the personal lives of people were affected but also way of making business, international political relations, multinational events etc. have changed dramatically.

In the same direction, after some regulations in aviation industry very a great competition had begun to be experienced among players of that industry field till present time among airline companies, airplane manufacturers and even governments.
Airline companies, that are directly related with the scheduling issue, can be categorized in different categories as:

International: Generally international airlines use airplanes over 140 seating capacity having ability to take people anywhere on the world. Majority of those types of companies produce $1 billion annually.

National: Most of the airline companies all over the world exist in that category in which 100-150 seated airplanes are operated. These companies produce revenue in a wide scale like $100 mio- $1 billion.

Regional: Comparing to national segment companies, firms in this category focus on short haul flights making less than $100 mio revenue.
Cargo: Companies focused on transporting goods are cargo companies becoming the basis of the worldwide commerce web.

On the other hand some significant issues and parameters which airline industry faces and tries to optimize somehow can be listed as:

1. Structures of routes
2. Airport capacities
3. Aircraft capacities
4. Lease or buy plans (For aircraft)
5. Weather
6. Fuel Cost
7. Labor

1.2. Airline Flight Scheduling

The aircraft routing decision is one of the most important components in the overall flight scheduling process of developing a profitable operational timetable of flights for an airline company. The flight scheduling process consists of two phases: a schedule construction phase and a schedule evaluation phase. In the schedule construction phase, a set of aircraft routes and the frequency of service on each route are first determined to maximize the profit generated from the operation, while taking into account the traffic estimates and revenue for every origin-destination pair, aircraft characteristics and operating costs, and some operating restrictions. Secondly, the construction phase is completed by scheduling departure times and assigning aircraft to match the routing and frequency decisions. The resulting timetable is then examined by operating personnel for feasibility and other cost and performance considerations in the schedule evaluation phase. Any desired improvements are then fed back into the construction phase, and a revised set of routes and the associated frequencies are determined. The flight scheduling process iterates between these two phases until a satisfactory final timetable is obtained. (Etschmaier, M., & Mathaisel, D. 1984).

1.3 Aircraft Routing

There are several steps in aviation industry for planning. Aircraft Routing is one of these steps and will be discussed within this study. The aircraft routing problem is the name of problem which refers to assigning flight legs to aircrafts defined by unique tail numbers. This unique number is determined by airlines according to their policy. “For example; in USA, aircraft tail numbers consist of a prefix ‘N’ and five alpha/numeric characters. For instance N723TZ; N is the country code for USA, 723 is used to designate the particular aircraft, and TZ is the airline code for ATA, Air Transport Association of America. For other countries, the tail number typically consists of two characters designating the country, followed by three alpha/numeric characters”. (Airline Operations and Scheduling, Bazargan, 2004).

The Aircraft Routing Process is, at most airlines, only seen as a feasibility problem; as the goal is to find a feasible assignment of aircraft to flight legs. However, it can be driven by operational quality objectives in some cases. For instance, maximizing the number of long connections increases aircraft availability to handle incidents on the day of operations. Long connections at appropriate airports can also offer alternative slots for minor maintenance activities (Gabteni and Grönkvist , 2006).

On the other hand while assigning flight legs to aircrafts, there are some operational constraints to be considered such as maintenance and restriction constraints. “These operational constraints can be categorized in two groups: general constraints applying to subsets of aircraft, such as fleets and subfleets, and tail constraints, applying specifically to particular tail numbers. The latter can relate to any technical aspect, such as limited fuel capacity, noise level, or in flight entertainment system functionalities. These aspects make each aircraft unique in the way it can be operated. Pre-assigned activities, such as heavy maintenance, are a particular type of tail constraint. The tail constraints are the fundamental difference between Tail Assignment and Aircraft Routing (Gabteni and Grönkvist, 2006). In that sense, we are excluding tail constraints for our aircraft routing problem. Moreover there are objectives also which can be listed as maximizing revenue or minimizing operating cost that are the main goals of whole industry

2. LITERATURE REVIEW

Maintenance routing problem is about creating maintenance-feasible aircraft routes. It is a feasibility problem rather than an optimization problem. Gopalan and Talluri (1998, pp. 46) describe a system for maintenance routing implemented at USAir. The maintenance requirements are simplified to a restriction on each aircraft to return to a maintenance base every three days, and it is assumed that the lines of flying during daytime (LOFs) are fixed. This problem can be solved in polynomial time, but Talluri (1998, pp. 32-43**)** has shown that the three-day problem is a special case, and that the general *N*-day problem is NP-hard.

Through Assignment Model (TAM) is a financially faced problem: A through flight is a two-leg flight between two locations, via the hub, simply using the same aircraft on both routes. Through assignment is the problem of deciding which leg-to-leg connections are to be through flights in order to gain time and idle aircrafts.

Most Aircraft Routing approaches combine the maintenance routing aspect with a cost function, which is often through value based, but can also capture other aspects. Kabbani and Patty (1992) model the aircraft routing problem for American Airlines as a set partitioning problem, where each column represents a week-long aircraft route. This makes it possible to handle general maintenance constraints, but the drawback is long running times. (1997, pp. 46), Clarke et al solve an aircraft rotation problem for Delta Air Lines, building maintenance feasible routes while maximizing through values. They require all aircraft to fly the same cyclic route (rotation). They formulate the problem as a TSP with side constraints, and solve it with Lagrangian relaxation.

Barnhart et al. (1998, pp.220) solve a combined fleet assignment/aircraft routing problem by an approach based on maintenance feasible *strings* of activities, that are combined to create feasible routes, within a branch-and-price framework. Short-haul instances with up to 190 flights are solved successfully. Elf et al. (2003, pp. 675) propose an aircraft rotation planning model for minimizing delay risk. In their model, a `delay risk' is either individual connections being too short, or consecutive visits to certain airports. Maintenance is not considered in their model, and a solution method based on Lagrangian relaxation is proposed. The existing literature does not consider individual aircraft requirements, and is typically based on cyclic models rather than dated ones. Many references on integrating constraint programming and column generation take the approach of solving the master problem with standard Linear Programming techniques, and use constraint programming to solve the often complex pricing problem. For example, Fahle et al. (2002, pp. 59) on the Airline Crew Rostering problem, and Rousseau et al. (2002**)** on the Vehicle Routing Problem (VRP). The latter shows promising results on some of the well-known Solomon instances. Caprara et al. (1998, pp. 76.) describes a combined CP/OR Crew Rostering application at the Italian State Railway. Their main solution method is constraint programming, and they use a Lagrangian relaxation to obtain lower bounds. Constraint programming is simply the idea of solving the relations between variables by stating them as constraint forms.

Literature on the aircraft routing problem with a fixed schedule spans over several decades, including the most recent one that is written by Subramanian et al. (1994, pp.104–120) Formulations and solution approaches are very similar. They rely on a mixed integer multicommodity network flow formulation based on a time-space graph representation that is solved by classical branch-and-bound. Barnhart et al. (1998 pp. 208-220) also solves the fixed schedule version but the authors introduce maintenance constraints in the model. A branch-and bound approach is used to solve it. Each node of the search tree corresponds to the linear relaxation of a set partitioning problem solved by column generation, where the column generator is a shortest path problem. Columns in the set partitioning problem refer to feasible aircraft itineraries. Desaulniers et al. (1997, pp. 841-855) introduces time windows on flight departures for the aircraft routing problem**.** The multi-commodity model now involves time variables. It is also solved by branch-and-bound and column generation except that the column generator is a time constraint shortest path problem (Desrosiers and Solomon 1995) .In Rexing et al. (2000, pp. 1-20), time windows are discretized, hence creating copies of each flight in the underlying graph representation. The column generator turns out to be a shortest path problem on an a cyclic graph. The model proposed in this paper for the aircraft routing problem as described in the previous section follows the general vehicle routing and crew scheduling framework presented in Desaulniers et al. (1998). As above, multi-commodity flows, branch-and-bound and column generation are used, one difficulty being the generation of the columns, or equivalently, the feasible aircraft itineraries. This is done using a specialized time constrained shortest path problem involving time window restrictions and linear node cost functions to account for flight spacing constraints as well as time dependent profit estimations. Ioachim et al. (1998, pp. 193-204) proposes an efficient dynamic programming algorithm to solve that type of constrained shortest path problem. It has already been used in several applications, among them are aircraft routing with schedule synchronization and simultaneous optimization of flight and pilot schedules in a recovery environment (Stojkovic M. et F. Soumis 2000-01). In the former application, flights on certain O-D pairs must be scheduled at the same time but on different days of a weekly horizon. In the later, small schedule perturbations keep aircraft itineraries the same but flight departure times are modified at a certain cost. These two applications and the one proposed in this paper have in common the fact that linear cost functions are associated with the time variables.

The fleet assignment model is formulated by Hane et al. [1995] as a multi-commodity flow problem. It is called leg-based because revenue effects between flight-legs are not modelled. To take such network effects into account, Barnhart et al. [2002] describe an enhanced model using demand forecasts for origin-destination pairs. An integrated model for schedule design and fleet assignment is presented by Lohatepanont and Barnhart (2004). They use the origin-destination fleet assignment model and flights are chosen from a 3 optional set of flights to maximise profit. The aircraft routing problem has been addressed in a number of publications, for example in Clarke et al. (1997), Feo and Bard (1989), Daskin and Panayotopoulos (1989), Gopalan and Talluri (1998), and Grönkvist (2006). Sarac et al. (2006) consider the aircraft routing problem on an operational level rather than a planning level. After fuel costs, crew salary is the second largest operational cost an airline has to account for. Therefore finding a minimal cost solution to the crew pairing problem is important. Indeed, it is a very difficult problem due to the large number of possible pairings, the complicated rule structure and the necessity to find integer solutions. For these reasons the crew pairing problem has received a lot of attention in the literature, Barnhart et al. (2003) for a detailed description of the crew pairing problem and a review of the literature addressing the problem. Also recently, Gopalakrishnan and Johnson (2005) give a comprehensive overview on state-of-the-art methods to solve the crew pairing problem. The crew pairing problem can be viewed as a separate optimisation problem with no effect on the cost of the integrated solution. It is referred to Ernst et al. (2004) for an annotated bibliography of rostering problems.

The literature reports on the integration of various combinations of airline scheduling problems. Klabjan et al. (2002) partially integrate aircraft routing, crew pairing and schedule design. They reverse the order of the crew pairing and aircraft routing problems. Plane count constraints are added to the crew pairing problem to guarantee the existence of a feasible solution for the aircraft routing problem by ensuring that at most the number of available aircraft is used at any time. Their results are based on a hub-and-spoke network. In this network only large airports (hubs) are linked by direct flights and all smaller airports (spokes) are only connected to one hub. Many aircraft meet at the same hub at the same time ensuring the existence of many feasible connections. This property leads to a much larger number of feasible routings than in an interconnected network. In interconnected networks many airports are linked with multiple other airports by direct flights. To include schedule design, the departure time of each flight is allowed to vary in some time window. This is done by relaxing feasibility parameters in the crew pairing problem and hence generating a larger set of pairings. Klabjan et al. (2002) solve the crew pairing problem via a linear programming based branch-and-bound algorithm.

Another model to integrate aircraft routing and crew pairing is proposed by Cordeau et al. (2001) and also by Mercier et al. (2005). They use Benders decomposition and branch-and-price tree model to solve the model. Employing the crew pairing problem as the sub problem as well as the master problem has been tested, the latter with better success. Both approaches add inequalities to the set partitioning polytopes of the problems. Cordeau et al. (2001) also reverse the sequential approach and try to solve the crew pairing problem first followed by the aircraft routing problem as in Klabjan et al. (2002). They apply this approach to an interconnected network but are not successful in obtaining feasible solutions for the aircraft routing problem. Cohn and Barnhart (2003) also integrate aircraft routing and crew pairing. They extend the crew pairing problem by using the aircraft routing problem as a second column generator next to the crew pairing generator. For each solution of the aircraft routing problem one variable is added to the crew pairing problem and a convexity constraint ensures the selection of one of the aircraft routing solutions in the final solution of the problem. LP based branch-and-price is used in this computationally expensive solution method. Mercier et al. (2005) find that their Benders decomposition approach yields better solutions in less computation time than the extended crew pairing model of Cohn and Barnhart (2003).

Sandhu and Klabjan (2007) partially integrate fleet assignment, aircraft routing, and crew pairing with a similar approach as Klabjan et al. (2002) and solve the model with both Lagrangian relaxation and Benders decomposition. Papadakos (2007) integrates the fleet assignment, aircraft routing, and crew pairing problems as an extension of the model of Mercier et al. (2005). Very recently, Mercier and Soumis (2007) extend their model (Mercier et al. (2005) and integrate aircraft routing and crew pairing with time windows for the departure times. Flights are allowed to depart five minutes earlier or later than originally scheduled. Binary variables are used to indicate which departure time is assigned to a flight. Equality constraints sum up the binary departure time variables for the crew and aircraft solutions and ensure that the same departure times are used in the solutions of both problems. Again, the authors use Benders decomposition to solve the problem. Models that focus purely on minimising cost tend to generate solutions that appear brittle in operations.

Such solutions incur large recovery costs once disruptions occur. In order to improve the behaviour in operations, a number of robustness measures have been introduced. Schaefer et al. (2005) uses expected operational cost for the crew pairings instead of planned cost. Interactive effects between pairings are ignored and a pushback strategy for recovery is used by him. In this strategy the flights are delayed until crew and aircraft are available. The authors estimate the costs and evaluate the quality of their solutions with SimAir, a Monte Carlo simulation of airline operations, see Rosenberger et al. (2002).

Yen and Birge [2006] formulate the crew pairing problem as a stochastic programming problem that they solve in a computationally expensive approach. Crew switching aircraft are penalised in the objective function. A similar measure of robustness is introduced by Ehrgott and Ryan (2002) in a deterministic approach. Crew pairings are penalised if crew are changing aircraft and the sit-time of the crew is less than the minimal sit-time plus some measure of delay of the incoming flight. Crew who stay on the same aircraft are not penalised. Thus, crew connections where disruptions are likely to propagate onto multiple flights are penalised. Robustness is treated as a second objective function in a bi-criteria approach. Mercier et al. (2005) also penalise crew changing aircraft on restricted connections.

Recently, Shebalov and Klabjan (2006) solve the crew pairing problem first and then maximise the number of move-up crews, i.e. crew that potentially can be swapped, without increasing the planned cost too much. They compare their method with the method of solving the standard crew pairing problem by simulating disruptions and find solutions with lower operational costs if the additional cost allowed for move-up crews is not too high.

A direct comparison between the various approaches is difficult due to different levels of integration, robustness measures and characteristics of schedules and rule-sets used. Cordeau et al. (2001) and Mercier et al. (2005) find their Benders decomposition approach superior to two recent models (Klabjan et al. 2002) and Cohn and Barnhart [2003]). It is also evident that a direct solution approach to the integrated model for large scale practical problems is much more time consuming than decomposition techniques if not intractable (Cordeau et al. (2001).

3. PROBLEM FUNDAMENTALS

This study is about ARP, Aircraft Routing Problem, which is briefly the challenge for finding an optimal flight path among cities having origin and destination pairs whether as direct routes or indirect routes through other cities.

The main objective is to minimize the total cost of reassigning of flights into routes by airplane companies. The goals are to cover all the flight by limited fleet with the maximization of maintenance opportunities of aircrafts within the time period. (Section 4.2.2.)

Possible contributions are to focus on the significance of this detailed issue among all other scheduling problems and solution methods of airline companies like THY and also to formulate a solution to THY based on their real data. (Section 4.1.)

3.1. Possible constraints in ARP

3.1.1. Connection Constraint:

Connecting flight legs is one of the basic constraints of aircraft routing problem since the time between arrival and departure times of an aircraft has a restricting specification. Because the preparation of the aircraft for new flight (cleaning, changing fleet and passengers) have to be considered. So it can be talked about a minimum connection time between arrivals and departures of an individual aircraft to satisfy needs. That minimum time concepts can be affected by various effects such as aircraft type, regulative applications, type of airport, and type of flight. For regulative applications schengen convention can be demonstrated as an example since in countries involved in convention have different applications for passenger which may change time for passenger’s transfer between aircrafts. Also type of airports affects minimum time because of the reason that transfer of passengers between gates in busy and big airports may take longer. Moreover minimum time changes by type of aircraft types and ages since their maintenance, refuelling and cleaning time may change.

3.1.2. Maintenance Constraint:

There are different maintenance types for different aircrafts. Maintenance activities are backbone of a successful and profitable airline company. In the airline industry, the role of maintenance is to provide safe, airworthy on time aircraft everyday. An airline generally has a diverse fleet of aircraft. Each fleet type has predetermined maintenance program established by the aircraft manufacturer and the Federal Aviation Administration (FAA). Aircraft maintenance must be planned and performed according to the prescribed procedures and standards.

FAA mandates that airlines perform four types of aircraft maintenance, commonly referred to as A- , B- , C- and D- checks. These checks vary in scope, duration and frequency. The most common maintenance check is the A- check, which involves a visual inspection of major systems. The FAA mandates that Check A is performed in every 60 flight hours. That is equivalent to five or six operating days. If an aircraft does not receive an A- check, that aircraft is grounded until it receives A-check. B- checks involve a through visual inspection and lubricating all moving parts. This type of maintenance is performed every 300 to 600 hours of flight. C- and D- checks involve taking the aircraft out of service, and are performed every one to four years.

The airline maintenance practices are generally more stringent. They perform A-checks every 3 to 4 days. The time required to perform an A- check on an aircraft is about 3 to 10 hours. The A- checks are normally performed between 10 pm and 8 am while the aircraft is on the ground. Therefore the aircraft routing problem must ensure that the aircraft is at the right base at the right time for this maintenance. Most aircraft routing models incorporate these A- checks in their formulations since they are routine.

Name	Period	Type	Name	Period	Type
100H	100	Flight hours	3C	18000	Flight hours
A	500	Flight hours	3C	1642	Days
2A	1000	Flight hours	E4	9000	Cycles
6A	3000	Flight hours	E4	1642	Days
C	6000	Flight hours	E5	12000	Cycles
C	547	Days	E5	2190	Days
2C	12000	Flight hours	L1	1000	Flight hours
2C	1095	Days	L2	3000	Flight hours

Table 3.1 .Maintenance constraints for a certain type of Boeing aircraft for an example airline. Observe that all checks are not included here, and the types and intervals might differ between airlines, and also depending on which maintenance program is implemented.

In addition to maintenance types, number of hangars in an airport also is a restricting factor of maintenance opportunities. The available maintenance opportunities are the number of hangars available on that airport.” (The Tail Assignment Problem, Matias Grönkvist, August 2005)

3.1.3. Flight Restriction Constraint:

Flight restriction constraints are based on restricting an individual aircraft flying to an individual destiny. There may be several reasons for that restriction. For example, the airport of destiny may forbid the aircraft which is to ground because the aircraft is too noisy. Another example can be that aircraft’s reverse thrusters may result in destructive happenings so it can be forbidden to ground for that aircraft. Moreover, fuel capacity of an aircraft is another factor on determining using that aircraft or not. For a long journey a long range aircraft must be used but not a low fuel capacitated aircraft. Flight restriction involves all these restrictions and these restrictions valid for different fleet types or aircrafts in same fleet.

3.2. Optimization Criteria

The optimization criterion in aircraft routing is often related to the robustness or quality of the solution, rather than real monetary costs. As an example, a typical objective function (*cost function*) rewards short and very long connections between successive activities, but penalizes medium length connections. The reason is that a medium length connection, i.e. a connection lasting between, say, two and three hours, causes an aircraft to be unavailable during a period of time. The aircraft cannot be used for anything while it is waiting, and perhaps it even has to wait at the gate, incurring a cost. On the other hand, if the connection is longer it is possible to use the aircraft as *standby*, i.e. letting it perform extra activities in the event of disruptions. Connections longer than some limit, for example two hours, are sometimes called *standby connections*. The standby time limit might vary from airport to airport, and also between different times of the day. Aircraft *availability* can loosely be defined as the proportion of non-flying time spent on standby connections. An alternative objective is to maximize the through values, i.e. the desirability of one-stop service between a pair of cities .For the interaction with crew pairing; a cost function rewarding the use of tight crew connections is used.

Finally, in case the aircraft routing problem is re-solved close to the day of operation, it might be desirable to obtain a new solution which differs as little as possible from the currently published solution, but which e.g. satisfies additional constraints. This can easily be modelled by adding penalty costs for all connections which are not in the current solution.

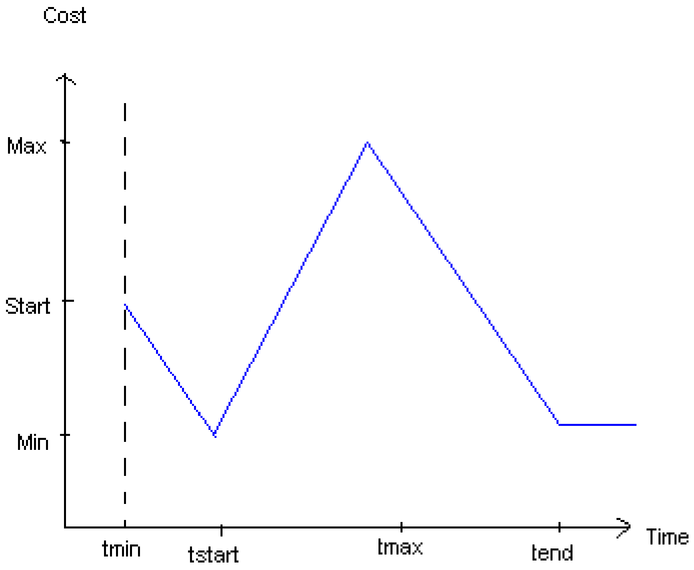

Figure3.1. The minimum connection time is denoted by *tmin*, so there are in fact no connections with connection time less than *tmin*. Connections close to the minimum connection time are penalized, up to an optimal connection time denoted *tstart*. Connections between *tstart* and *tmax* in duration are penalized with a linearly increasing penalty, and connections longer than *tmax* are penalized with a linearly decreasing penalty.” (The Tail Assignment Problem, Matias Grönkvist , August 2005)

4. DATA and METHODOLOGY

4.1 Problem Description

Here in order to demonstrate the solution method for Aircraft Routing Problem, our problem is implemented on Lingo Optimization Software. In order to base our problem on the real-life circumstances, data of problem which is going to be focused on is gathered from Turkish Airlines Official Site. Official timetable is simplified by eliminating flights to a considerable number to be solved.

Flight No	**Origin**	**Departure time**	**Destination**	**Arrival Time**	**(Hrs.)**	**Fleet Type**
TK 451	ADA	05:00	IST	06:30	1,5	737 CY
TK 105	ANK	05:10	IST	06:10	1	737 CY
TK 104	IST	06:15	ANK	07:15	1	737 CY
TK 482	ANK	06:30	ADA	07:30	1	737 CY
TK 453	ADA	07:00	IST	08:30	1,5	737 CY
TK 111	ANK	08:00	IST	09:00	1	737 CY
TK 483	ADA	08:15	ANK	09:15	1	737 CY
TK 116	IST	09:00	ANK	10:00	1	737 CY
TK 120	IST	11:00	ANK	12:00	1	737 CY
TK 459	ADA	11:10	IST	12:45	1,58	737 CY
TK 462	IST	11:15	ADA	12:50	1,58	737 CY
TK 124	IST	13:00	ANK	14:00	1	737 CY
TK 127	ANK	13:00	IST	14:00	1	737 CY
TK 464	IST	14:15	ADA	15:50	1,58	737 CY
TK 130	IST	16:00	ANK	17:00	1	737 CY
TK 135	ANK	17:00	IST	18:00	1	737 CY
TK 138	IST	17:30	ANK	18:30	1	737 CY
TK 096	IST	19:10	ADA	20:45		737 CY
TK 484	ANK	19:30	ADA	20:30	1	737 CY
TK 467	ADA	19:40	IST	21:10	1,5	737 CY
TK 470	IST	19:45	ADA	21:20	1,58	737 CY
TK 147	ANK	20:00	IST	21:00	1	737 CY
TK 485	ADA	21:15	ANK	22:15	1	737 CY

TK 471	ADA	22:10	IST	23:45	1,58	737 CY
TK 155	ANK	23:00	IST	00:00	1	737 CY
TK 472	IST	23:35	ADA	01:10	1,58	737 CY
TK 168	IST	23:45	ANK	00:45	1	737 CY

Table 4.1

Above, the data which are going to be used for problem is seen. In that timetable, there are 3 cities which are ADA for Adana, ANK for Ankara and lastly IST for Istanbul. Istanbul is the hub of that network and so all the maintenance facilities are located in Istanbul.

In order to define problem formally, firstly all assumptions are listed related to the operation of the system:

4.1.1. Assumptions:

1. Every aircraft have to gain maintenance service once in two day route which means every aircraft must overnight in Istanbul in 2 day period.
2. Turn-around times are determined to be 45 min.
3. Routes are considered as closed routes which mean that routes end cities which they started.
4. Number of aircrafts to be determined according to result of model in order to reach a feasible solution.
5. All aircrafts are in same fleet type.

4.1.2. Possible Routes

After those assumptions problem can be modelled. But one must consider that solution of problem consists of two phases. In the first phase all feasible routes must be calculated. According to assumptions, a feasible route which has 2 day period is made and between each flight leg there is a 45 minute difference referring to turn-around times.

Three feasible routes can be seen below.

Routes	**Day 1**	**Day 2**
Route 1	**TK104-**TK484-**TK485**	**TK482**-TK483-**TK155**
Route 2	**TK459-**TK464-**TK471**	**TK116-**TK135-**TK470**
Route 3	**TK104-**TK111-**TK462-**TK467-**TK168**	**TK105-**TK116-**TK127-**TK130-**TK484-**TK471

Table 4.2

In above candidate routes, it can be easily realized that first route starts with flight number TK104 which is the flight leg between Istanbul-Ankara with departure time 06:15 and arrival time 07:15.Just after that flight, TK484, which is Ankara-Adana, comes and its departure time is 19:30 and arrival time is 20:30. All specifications of that combination suits to constraints considered. All other routes have same appropriateness. Before using them in the model, all possible routes must be determined by using software. In that project, because of lack of that type of software, 30 of possible routes are determined manually which can be seen below:

Routes	**DAY 1**	**DAY2**	**Maintenance Opportunity**
1	451-116-127-130-484-485-155	104-111-120-127-138-147-472	1
2	104-484-485	482-483-155	1
3	104-135-472	459-464-471	1
4	111-120-127	116-484-485	1
5	453-120-127-130-147	462-467-472	1

6	116-484	483-127-130-147	1
7	105-116-127-130-484-485-155	104-111-462-467-168	1
8	111-462-471	104-111-120-127-130-147-168	1
9	111-124-135-96-471	104-111-124-135-168	1
10	104-127-130-484	459-464-467	1
11	482-483-484-471	116-127-130-484-485	1
12	459-464-471	116-135-470	1
13	104-111-462-467-168	105-116-127-130-484-471	1
14	105-116-127-96-471	483-135-470-471	2
15	453-120-127-130-147	462-467-472	1
16	459-464-471	104-111-120-127-96	1
17	482-483-135-96-471	104-111-120-127-130-147-168	1
18	462	471	1
19	104-484	483-155	1
20	482-467	462-485	1
21	483-127	120-484	1
22	105-116-484-471	104-111-462-485	1
23	451-116-127	464-467-472	1
24	116-135-96	453	1
25	451-120-127	104-111-470	1
26	453-120-127-130-147	464-467-472	1

27	105-116-135	120-484-485	1
28	105-124-135	124-135	1
29	453-464	453-464-467	1
30	482-459	168	1

Table 4.3

4.2. Integer Modelling

4.2.1. Decision Variable

As it is known, aim of the problem is assigning routes to aircrafts. Since possible routes were determined before, they are all candidates for assigning in real timetable. So decision variable in that problem is X_j which refers to candidate route j. Moreover it is binary decision variable. If the route is selected and assigned as a real route in the timetable, then its value is directly 1. In reverse, value “0” tells that the route is not used in the schedule.

Xj=1 if route j is selected, j=1,2,3,.....,30
0 otherwise

4.2.2. Objective Function

In our Aircraft Routing Problem, main goal is maximizing maintenance chance for aircrafts. Maintenance opportunity means overnight in Istanbul. Let the number of maintenance opportunity for route j be m_j.

Each routes that are placed inside the schedule (*Xj=1)* is multiplied by its maintenance value and the sum of this calculation gives us the value that we are looking for to maximize.

Then objective function is:
Maximize $\sum m_j x_j$.

4.2.3. Constraints

The problem is limited by flight amount of each aircraft (flight coverage) and possible amount of aircrafts for assigning in the fleet (fleet capacity) which form the two constraints.

Flight Coverage

As it can be seen in sample routings, there are several flights in each route. Our aim is to cover all flights in initial timetable. In order to cover all flights constraints below must be used:

$X1+X3<=1$ according to sample routes. In that constraint it is assured that flight TK104 is covered at least once. Like that constraint, constraints for each flight must be entered.

Fleet Capacity

Another constraint in the problem is fleet capacity constraint. In other words, routes chosen can not exceed the number of aircrafts airline has in its fleet. Then constraint is converted to mathematical demonstration as:
$\sum Xj <=$ Number of aircrafts in fleet.

4.2.4. Lingo Model

In order to solve the problem by Lingo, model must be converted into appropriate Lingo syntax. First of all, “sets” of routes and flights must be built. By using sets, constraint and data about routes and flights can easily be written.

```
SETS:
ROUTE/1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30/:SELECT,
MAINTENANCE;
FLIGHT/1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27/;
LINKS(FLIGHT,ROUTE):COVER;
END SETS
```

As it can be seen above, there are 3 sets. First one is ROUTE set. It refers to all 30 routes determined before. Moreover the first set defines attributes of routes as SELECT (Because routes are selected as it was discussed earlier) and MAINTENANCE (It refers to maintenance opportunities for a route).The second set is FLIGHT set. Like ROUTE , it refers to flights in timetable.

Last be not least important set is LINKS set. That last set supplies the relationship between FLIGHT and ROUTE sets. The whole row means "There is a coverage relationship between flights and routes." Detailed data of those routes are written in DATA stage.

After defining sets, objective function must be defined like shown below;

```
!OBJECTIVE FUNCTION;

MAX=@SUM(ROUTE(J):MAINTENANCE(J)*SELECT(J));
```

The above objective function, which is formulated as *Maximize* $\sum m_j x_j$ in Section 4.2.2, maximizes maintenance chance by forcing to select routes with high maintenance opportunities. Because, as it can be easily realized, that operation can be interpreted as "summation of value of Select attribute of Route j". Logically, value of Select attribute of a route is 0 or 1. So program will assign 1 to routes with higher maintenance opportunities.

Next part is the definition of constraints. Constraints of our problem in Lingo language can be seen below:

```
!CONSTRAINTS;

@FOR(FLIGHT(I):@SUM(ROUTE(J):COVER(I,J)* SELECT(j))<=1);
@SUM(ROUTE(J): SELECT(J))< = 8
@FOR(ROUTE(J):@BIN (SELECT(J)));
```

The first constraint (first line) is Coverage of all flights. LINK set which we defined in SETS part is used. Attribute of that set was COVER and interpretation of line is for every i, summation of value of flight i in route j is equal or greater than 1.Cover value of a flight in a route is 0 or 1. For example; Let Flight 484 be only in both Route 5 and Route 7.If one or both of these flights is not selected Flight 484 will not be covered. So sum of COVER value of these routes must be equal or greater than 1.

Constraint in second line is the capacity of the fleet size. In other words, number of selected routes can not exceed number of aircrafts in our fleet. Sum of values of SELECT attribute of ROUTE set is equal or greater than fleet size. There is point to be considered in aircraft number which will be discussed in Results part. (Section 5) Last constraint is defining SELECT attribute of Route set as binary variable which was mentioned before.

Next step after completing definition of sets, objective function and constraint is defining data of problem. Data which have to be defined are about Maintenance attribute of Route and Cover attribute of LINK set. (See Appendix A)

First line shows us the maintenance opportunities for all routes.

```
MAINTENANCE=1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1;
```

There are 30 routes and so 30 values. For example number of maintenance opportunities in route 1 is 1 which is shown in first place and it goes to 30 in ascending order of indexes of routes.

Second data set is a matrix whose horizontal values refer to routes and vertical values are flights. As it can be seen values are binary which is defined as COVER attribute of LINK set between routes and flights. (Also see Appendix A)

COVER= 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0

Value is 1 if that particular route covers particular flight, 0 otherwise. For example in order to check whether route 6 covers flight 7, column 6 should be found for route 6 and row 7 for flight 7. It can be seen that value is 1 so it can be said that route 6 covers flight 7.

5. RESULTS and DISCUSSION

After completing Lingo model it can be solved. Software runs on a computer which is Intel Pentium 1.73 GHz with 512 RAM on it. Solution of problem took 1 second.

Result report is as below. (See Appendix B)

```
Global optimal solution found.
   Objective value:                          7.000000
   Extended solver steps:                           0
   Total solver iterations:                       7
```

It can be seen which routes are selected from Select value with bold font.

For instance;

```
SELECT( 1)       1.000000          -1.000000
SELECT( 5)       1.000000          -1.000000
```

According to these results Routes 1, 5, 9, 11, 12 and 14 are selected. Let’s check whether these routes cover all flights or not:

Routes	DAY 1	DAY2	**Maintenance Opportunity**
1	451-116-127-130-484-485-155	104-111-120-127-138-147-472	1
5	453-120-127-130-147	462-467-472	1
9	111-124-135-96-471	104-111-124-135-168	1
11	482-483-484-471	116-127-130-484-485	1
12	459-464-471	116-135-470	1
14	105-116-127-96-471	483-135-470-471	2

Table 5.1

As it can be seen, selected routes cover all flights in initial timetable. So it can be said that result is feasible and optimum. Moreover value of objective function is 7. It means that our fleet of six aircrafts may have the total of seven times (1+1+1+1+1+2 as shown in Table 3.4's last column) of opportunity for maintenance in 2-day period.

Results can be demonstrated in terms of cities as below:

Routes	**Day 1**	**Day 2**
1	ADA-IST-ANK-IST-ANK-ADA-ANK-IST	IST-ANK-IST-ANK-IST-ANK-IST-ADA
5	ADA-IST-ANK-IST-ANK-IST	IST-ADA-IST-ADA
9	ANK-IST-ANK-IST-ADA-IST	IST-ANK-IST-ANK-IST-ANK
11	ANK-ADA-ANK-ADA-IST	IST-ANK-IST-ANK-ADA-ANK
12	ADA-IST-ADA-IST	IST-ANK-IST-ADA
14	ANK-IST-ANK-IST-ADA-IST	IST-ADA-ANK-IST-ADA-ANK

Table 5.2

It can be easily realized that routes are closed routes each ending in the city it starts. Moreover aircrafts stay overnights in cities which they landed in last flight of day. Next day they start their remaining part of routes from the same city.

Fleet capacity in that problem is assumed as 6 aircrafts. However, solutions with different number of aircrafts may be possible in restriction "fleet size>=6". If it was tried with fleet capacity less than 6 aircrafts software would not give a feasible solution. So adjustment of fleet capacity constraint according to the minimum value, leads us to a feasible result.

In addition, constraints and objective function of that problem may vary according to needs of user. For example objective function may be minimization of operating cost of aircrafts if assumption 5 is not made. Another example of an objective function is balancing utilization of aircrafts. In other words minimizing difference between durations of routes assigned to each aircrafts. If that was objective function in that project, duration of each flight leg would be input data for us.

6. CONCLUSION

The study starts with an introduction section that presents us the airline industry and the upcoming significance of aircrafts since World War I. The airline company types are introduced and industry's main concerns like capacities are listed. The basic concept of the study is mentioned as Aircraft Routing Problem is the problem that refers to assigning flight legs to aircrafts designed by unique tail numbers.

Second section consists of the literature review and shows hints about our study and problem. We see that many papers have been focused on this and similar problems in order to contribute to airline industry.

Beginning from the third section our problem is presented, modelled, solved and discussed. The main objective is to minimize the total cost of reassigning of flights into routes by airplane companies. Our aim is to create a feasible solution for ARP for a real-life situation based on the data that is gained by THY official website. Our model's objective function is $Maximize \sum m_j x_j$ which is the maximum multiplication of two variables: maintenance opportunity of the route m_j and the binary value of whether the route is assigned or not x_j. Besides, two constraints are mentioned as fleet capacity must be larger than the total of binary positive routes of x_j.

Our data is presented and solved according to Adana, Istanbul and Ankara flights of Turkish Airlines. And also specific flights have to be covered like $X1+X3=1$. Lingo Software's findings are shown in Appendices as the result 7 is discussed in the previous Section 5. The six routes that are found to be valued as 1 by Lingo form our whole package of routes which are checked if they consist all 30 routes and resulted positively. We have narrowed down our 30 routes into 6 packages of takeoffs and total of seven maintenance sessions are held for aircrafts for minimum time loss.

To sum up, we have created a model by two simple constrains and one objective function in order to formulate a solution to a problem which can be used within the policy of airplane firms for reducing their cost by Lingo Software.

```
SETS:
ROUTE/1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30/:SELECT,
MAINTENANCE;
FLIGHT/1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27/;
LINKS(FLIGHT,ROUTE):COVER;
END SETS
!OBJECTIVE FUNCTION;
MAX=@SUM(ROUTE(J):MAINTENANCE(J)*SELECT(J));
 !CONSTRAINTS;
@FOR(FLIGHT(I):@SUM(ROUTE(J):COVER(I,J)* SELECT(j))=1);
@SUM(ROUTE(J): SELECT(J))< = 8;
@FOR(ROUTE(J):@BIN (SELECT(J)));
```

```
Global optimal solution found.
  Objective value:                    7.000000
  Extended solver steps:                     0
  Total solver iterations:                   7
```

As shown above, another conclusion also shows us the syntax of our modelling and implementation steps for the problem.

Appendix A

```
DATA:
MAINTENANCE=1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1;
COVER= 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0
0 0 0 0 0 0 1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 0 1 1 0 0
1 1 1 0 0 0 1 1 1 1 0 0 1 0 0 1 1 0 1 0 0 1 0 0 1 0 0 0 0 0
0 1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 1
0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 1 0 1 0 0 1 0
1 0 0 1 0 0 1 1 1 0 0 0 1 0 0 1 1 0 0 0 0 1 0 0 1 0 0 0 0 0
0 1 0 0 0 1 0 0 0 0 1 0 0 1 0 0 1 0 1 0 1 0 0 0 0 0 0 0 0 0
1 0 0 1 0 1 1 0 0 0 1 1 1 1 0 0 0 0 0 0 0 1 1 1 0 0 1 0 0 0
1 0 0 1 1 0 0 1 0 0 0 0 0 0 1 1 1 0 0 0 1 0 0 0 1 1 1 0 0 0
0 0 1 0 0 0 0 0 0 0 0 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 1
0 0 0 0 1 0 1 1 0 0 0 0 0 0 1 0 0 1 0 1 0 1 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0
1 0 0 1 1 1 1 1 0 1 1 0 1 1 1 0 1 0 0 0 1 0 1 0 1 0 0 0 0 0
0 0 1 0 0 0 0 0 0 1 0 1 0 0 0 1 0 0 0 0 0 0 1 0 0 1 0 0 1 0
1 0 0 0 1 1 1 0 0 1 1 0 1 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 0 0 0 0 1 0 0 1 0 0 0 0 1 0 0 0 0 0 0 1 0 0 1 1 0 0
1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 1 0 0 0 0 0 1 0 1 1 0 0 0 0 0 0 0 1 0 0 0 0 0
1 1 0 1 0 1 1 0 0 1 1 0 1 0 0 0 0 0 1 0 1 1 0 1 0 0 0 1 0 0
0 0 0 0 1 0 1 0 0 1 0 0 1 0 1 0 0 0 0 1 0 0 1 0 0 1 0 0 1 0
0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0
1 0 0 0 1 1 0 1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0
1 1 0 1 0 0 1 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 1 0 0
0 0 1 0 0 0 0 1 1 0 1 1 1 1 0 1 1 1 0 0 0 1 0 0 0 0 0 0 0 0
1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
1 0 1 0 1 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 0 1 0 0 1 0 0 0
0 0 0 0 0 0 1 1 1 0 0 0 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 1 ;
END DATA
```

Appendix B

```
Global optimal solution found.
   Objective value:                    7.000000
   Extended solver steps:                     0
   Total solver iterations:                   7


                      Variable        Value       Reduced Cost
                    SELECT( 1)      1.000000         -1.000000
                    SELECT( 2)      0.000000         -1.000000
                    SELECT( 3)      0.000000         -1.000000
                    SELECT( 4)      0.000000         -1.000000
                    SELECT( 5)      1.000000         -1.000000
                    SELECT( 6)      0.000000         -1.000000
                    SELECT( 7)      0.000000         -1.000000
                    SELECT( 8)      0.000000         -1.000000
                    SELECT( 9)      1.000000         -1.000000
                   SELECT( 10)      0.000000         -1.000000
                   SELECT( 11)      1.000000         -1.000000
                   SELECT( 12)      1.000000         -1.000000
                   SELECT( 13)      0.000000         -1.000000
                   SELECT( 14)      1.000000         -2.000000
                   SELECT( 15)      0.000000         -1.000000
                   SELECT( 16)      0.000000         -1.000000
                   SELECT( 17)      0.000000         -1.000000
                   SELECT( 18)      0.000000         -1.000000
                   SELECT( 19)      0.000000         -1.000000
                   SELECT( 20)      0.000000         -1.000000
                   SELECT( 21)      0.000000         -1.000000
                   SELECT( 22)      0.000000         -1.000000
                   SELECT( 23)      0.000000         -1.000000
                   SELECT( 24)      0.000000         -1.000000
                   SELECT( 25)      0.000000         -1.000000
                   SELECT( 26)      0.000000         -1.000000
                   SELECT( 27)      0.000000         -1.000000
                   SELECT( 28)      0.000000         -1.000000
                   SELECT( 29)      0.000000         -1.000000
```

```
    SELECT( 30)        0.000000           -1.000000
  MAINTENANCE( 1)       1.000000           0.000000
  MAINTENANCE( 2)       1.000000           0.000000
  MAINTENANCE( 3)       1.000000           0.000000
  MAINTENANCE( 4)       1.000000           0.000000
  MAINTENANCE( 5)       1.000000           0.000000
  MAINTENANCE( 6)       1.000000           0.000000
  MAINTENANCE( 7)       1.000000           0.000000
  MAINTENANCE( 8)       1.000000           0.000000
  MAINTENANCE( 9)       1.000000           0.000000
 MAINTENANCE( 10)       1.000000           0.000000
 MAINTENANCE( 11)       1.000000           0.000000
 MAINTENANCE( 12)       1.000000           0.000000
 MAINTENANCE( 13)       1.000000           0.000000
 MAINTENANCE( 14)       2.000000           0.000000
 MAINTENANCE( 15)       1.000000           0.000000
 MAINTENANCE( 16)       1.000000           0.000000
 MAINTENANCE( 17)       1.000000           0.000000
 MAINTENANCE( 18)       1.000000           0.000000
 MAINTENANCE( 19)       1.000000           0.000000
 MAINTENANCE( 20)       1.000000           0.000000
 MAINTENANCE( 21)       1.000000           0.000000
 MAINTENANCE( 22)       1.000000           0.000000
 MAINTENANCE( 23)       1.000000           0.000000
 MAINTENANCE( 24)       1.000000           0.000000
 MAINTENANCE( 25)       1.000000           0.000000
 MAINTENANCE( 26)       1.000000           0.000000
 MAINTENANCE( 27)       1.000000           0.000000
 MAINTENANCE( 28)       1.000000           0.000000
 MAINTENANCE( 29)       1.000000           0.000000
 MAINTENANCE( 30)       1.000000           0.000000

Row    Slack or Surplus      Dual Price
                 1        7.000000           1.000000
                 2        0.000000           0.000000
                 3        0.000000           0.000000
                 4        1.000000           0.000000
                 5        0.000000           0.000000
                 6        0.000000           0.000000
```

```
 7    1.000000       0.000000
 8    1.000000       0.000000
 9    3.000000       0.000000
10    1.000000       0.000000
11    0.000000       0.000000
12    0.000000       0.000000
13    0.000000       0.000000
14    3.000000       0.000000
15    0.000000       0.000000
16    2.000000       0.000000
17    1.000000       0.000000
18    0.000000       0.000000
19    0.000000       0.000000
20    1.000000       0.000000
21    0.000000       0.000000
22    0.000000       0.000000
23    1.000000       0.000000
24    1.000000       0.000000
25    3.000000       0.000000
26    0.000000       0.000000
27    1.000000       0.000000
28    0.000000       0.000000
29    0.000000       0.000000
```

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